A Strategic-Equilibrium Based Equal-Opportunity for all Players Attractor

Equilibrio estratégico-basado en igualdad de oportunidades para todos los jugadores atractores

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ABSTRACT

The strategic equilibrium of an N-person cooperative game with transferable utility is a system composed of a cover collection of subsets of N and a set of extended imputations attainable through such equilibrium cover. The system describes a state of coalitional bargaining stability where every player has a bargaining alternative against any other player to support his corresponding equilibrium claim. Any coalition in the sable system may form and divide the characteristic value function of the coalition as prescribed by the equilibrium payoffs. If syndicates are allowed to form, a formed coalition may become a syndicate using the equilibrium payoffs as disagreement values in bargaining for a part of the complementary coalition incremental value to the grand coalition when formed. The emergent well known-constant sum derived game in partition function is described in terms of parameters that result from incumbent binding agreements. The strategic-equilibrium corresponding to the derived game gives an equal value claim to all players. This surprising result is alternatively explained in terms of strategic-equilibrium based possible outcomes by a sequence of bargaining stages that when the binding agreements are in the right sequential order, von Neumann and Morgenstern (vN-M) non-discriminatory solutions emerge. In these solutions, a preferred branch by a sufficient number of players is identified: the weaker players syndicate against the stronger player. This condition is referred to as *the stronger player paradox*. A strategic alternative available to the stronger players to overcome the anticipated not desirable results is to volun-

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tarily lower his bargaining equilibrium claim. In doing the original strategic equilibrium is modified and vN-M discriminatory solutions may occur, but also a different stronger player may emerge that has eventually will have to lower his equilibrium claim. A sequence of such measures converges to the equal opportunity for all vN-M solution anticipated by the strategic equilibrium of partition function derived game.
Key words: strategic equilibrium, syndicates, derived partition function form game, von Neumann and Morgenstern solutions, stronger player paradox, Equal opportunity, attractor.

RESUMEN

El equilibrio estratégico de un juego cooperativo con N personas con utilidad transferible es un sistema compuesto de una colección cubierta de subconjuntos de N y un conjunto de imputaciones extendidas, adquiriéndolo a través de cierto equilibrio. El sistema describe un estado de estabilidad de la negociación de coalición en donde cada jugador tiene una alternativa de negociación frente a cualquier otro jugador para apoyar su afirmación de equilibrio correspondiente. Cualquier coalición en el sistema de sable puede formar y dividir las funciones de valor característico de la coalición según estipula el equilibrio de pagos. Por ejemplo, si los sindicatos pueden formar una coalición, esto puede convertirse en un nuevo sindicato, usando los pagos de equilibrios como valores de desacuerdos en una negociación de una parte del valor creciente de la coalición complementaria a la gran coalición cuando ésta sea formada. El juego bien conocido como suma derivada de constantes en una función de partición se describe en términos o parámetros que resultan de títulos acuerdos vinculantes. La estrategia de equilibrio correspondiente al juego derivado da un valor igual a todos los jugadores. Este resultado es sorprendente. Alternativamente se explica en términos de equilibrio estratégico basado en los posibles resultados de una secuencia de etapas de negociación que cuando los acuerdos son vinculantes en el orden secuencial de la derecha von Neumann y Morgenstern (vN-M) soluciones no discriminatorias. Estas soluciones son una rama preferida por un número suficiente de jugadores que se identifica: el sindicato de los jugadores más débiles contra el jugador más fuerte. Esta condición se conoce como *la paradoja del jugador más fuerte*. Una alternativa estratégica a disposición de los jugadores más fuertes para superar los resultados esperados es reducir voluntariamente su solicitud de equilibrio de negociación. Para ello el equilibrio estratégico original es modificado y soluciones M vN discriminatorio se pueden producir, sino también un jugador diferente más fuerte puede surgir que eventualmente tendrá que bajar su demanda de equilibrio. Una secuencia de tales medidas converge a la igualdad de oportunidades para toda la solución vN-M previsto por el equilibrio estratégico de la función de partición derivado juego.
Palabras clave: equilibrio estratégico, sindicatos, función necesaria de un juego, soluciones de von Neumann y Morgenstern, paradoja del jugador más fuerte, igualdad de oportunidades, atractor.

I. INTRODUCTION

The strategic–equilibrium of a cooperative game is a fundamental characteristic of the game that describes the maximum sustainable claims in the bargaining process that takes place at the coalition formation level. In general is not to be considered a solution to the game but as a foundation or defining point for any possible solution to the game. It is a conditional system of possible mutually exclusive interrelated coalitions and corresponding sustainable payoffs. It is mathematically characterized in terms of balanced collections and utility transfers that relate to each other defining a strategic-equilibrium in conditions that can be clearly explained by means of classical theorems of the alternatives for matrices. Linearly balanced collections provide bargaining structures consisting of collections of coalitions where players have bargaining alternatives against each other.

This paper takes the class of general sum 3-person cooperative game with transferable utility that satisfies the triangular inequality. The explicit strategic equilibrium for these games is obtained. This equilibrium describes the maximum sustainable claims players can make at the coalition formation bargaining level. These claims are then used as disagreement payoffs in the syndicate formation bargaining level.

Two types of splitting rates are to be defined by binding agreements: (1) The syndicate internal agreement on how the gains from the syndicate negotiations are to be divided among its members and (2) The external syndicate agreement rate between the syndicate and the complementary coalition if the grand coalition is to form.

A system of bargaining departing points for each possible syndicate is obtained as an extension of the strategic-equilibrium system and a general description of strategic-equilibrium based possible outcomes is given in terms of the syndicate splitting rates mentioned above.

The parametric description of outcomes associated with the strategic-equilibrium and the syndicate possible gains allows us to obtain a derived game in partition function form.

Under certain minimal restrictions the derived game is triangular and the unique strategic equilibrium is obtained to give an equal value assignment to every player.

In considering a different course of analysis we may give a full description of possible outcomes by establishing the interrelated system of outcomes that may occur for given standards of the splitting agreement rates.

The resulting system of strategic-equilibrium based possible outcomes turns out to be a von Neumann and Morgenstern non-discriminatory solution if the binding agreements are established in a "the right" sequential order.

The obtained vN-M discriminatory solution, as a solution system, has a peculiar characteristic. One of the branches may be preferred by a sufficient number of players so as to be a dominant part of the "solution". This branch is the one where the weaker players syndicate against the stronger player. This condition has been given the name of "the stronger player paradox". A possible strategic response to avoid the paradox consists on lowering the maximum sustainable claim by the stronger player. In doing so vN-M discriminatory possible solutions emerge. However a possible sequence of similar moves by the new stronger players may emerge, converging to a bargaining attractor were an equal opportunity for all players is revealed. This attractor is anticipated in the strategic-equilibrium obtained for the derive in partition function game.

II. THE STRATEGIC-EQUILIBRIUM FOR THE 3-PERSON GENERAL-SUM GAME

Let us consider the 0-normalized 3-person general sum cooperative game with corresponding characteristic function:

$v(N)= v_N$, $v(\{i, j\}) = v_{ij}$, $v(\{i\}) = 0$; $i, j \in N=\{ 1, 2, 3\}, i\neq j$

Let W_{3x3} the characteristic matrix of the 2-person coalitions and w a 3-vector with characteristic function values $w = (v_{12}, v_{13}, v_{23})^t$. Without loss of generality we may assume $v_{12} \geq v_{13} \geq v_{23}$.

The following definitions introduce some of the basic concepts necessary to identify and characterize the strategic equilibrium for the specific given class of triangular cooperative games:

A 3-person general-sum game is said to be *triangular* if and only if $v_{12} \leq v_{13} + v_{23}$.

An extended-imputation is a non-negative 3-vector. That is, any x in E^3, $x = (x_1\ x_2\ x_3)^t$ with $x_i \geq 0$, $i = 1, 2, 3$.

A non empty collection $C = \{C_1, \ldots, C_{|C|}\}$ of subsets of N is said to be a *covering collection* or simply a *cover* of N if and only if for every $j \in N$ there exists $C_i \in C$ such that $j \in C_i$.

An extended imputation x is said to be *attainable* (through C) if there exists a *covering collection* C of N = {1, 2, 3} such that v(C) = x(C) for any $C \in C$. Note that if C = {N}, an extended imputation x attainable through N, that is, v(N) = x(N), is simply an imputation.

The added value, incremental value or marginal contribution of player k to the grand coalition value v(N) is defined to be

$$e_k = v_N - v_{ij} = v_N - v_{N-\{k\}}$$

The value level of an extended imputation is define to be

$$x(N) = \sum_{j=1}^{3} x^\circ_j = x_1 + x_2 + x_3$$

A cover collection C = $\{C_1, \ldots, C_k\}$ of N is said to be p-balanced if and only if there exist a positive k-vector $\gamma \in E^k$ such that $W^t \gamma = J$ here W is a k by n characteristic matrix of the collection C and $J = (1, 1, 1)^t$ is the unit 3-vector.

The *maximum sustainable claims*[1] the players can make based upon the characteristic function values if the corresponding 2-person coalition forms is given by the solution to the system of equations W x = w.

Since the inverse of W is

$$W^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \text{ and } W = \begin{bmatrix} v_{12} \\ v_{13} \\ v_{23} \end{bmatrix}$$

is the vector of characteristic function values, then extended imputation.

$x^\circ = W^{-1} w$ is given by:

$$x^\circ_1 = ½ (v_{12} + v_{13} - v_{23})$$

$$x^\circ_2 = ½ (v_{12} - v_{13} + v_{23})$$

$$x^\circ_3 = ½ (-v_{12} + v_{13} + v_{23})$$

Note that[2]

$$x^\circ_i + x^\circ_j = v_{ij} \quad i, j = 1,2,3, \; i \neq j$$

and

$$x^o(N) = \sum_{j=1}^{3} x^o_j = \frac{1}{2}(v_{12} + v_{13} + v_{23})$$

Since the game is triangular the resulting vector is a unique *attainable extended imputation* that admits no transfer of utility[3] among players.

[2] Games where there is an imputation that satisfies $x_i + x_j = v_{ij}$, i, j =1, 2, 3, $i \neq j$ are called quota games. The existence of the strategic equilibrium makes all triangular 3-person cooperative game a quota game.

[3] In the utility transfer analysis chapter, admissible transfers are analyzed to characterize strategic stability in terms of balanced collections.

That is, if a player h ask his potential partner k, e units so that the resulting claims would be $x°_h + e$, $x°_k$ -e, player k would point out that to player h that for him to obtain the claim $x°_h$ + e, elsewhere he would have to offer player l, $x°_l$ -e while he player k could protect his claim $x°_k$ and at the same time he could give player l the full amount of his sustainable claim $x°_l$.

The extended-imputation vector x° will be referred as the vector of maximum sustainable claims since no higher claims may be protected under all possible circumstances.[4]

The system of claims and the corresponding cover of N that supports them conform the pair (x°, C) where C = {{1, 2}, {1, 3}, {2, 3}}. This pair will be referred as the *fundamental strategic equilibrium* of the game. In general the vector x° is not a solution[5] but rather may well be considered as the strategic base or fundamental attractor around which the solutions of the 3-person general sum game emerge.

The value level of x° namely x°(N) will be referred to as *the strategic-equilibrium value level of the game.*

The cover C = {{1, 2}, {1, 3}, {2, 3}} is p-balanced with balancing vector $\gamma = (1/2,\ 1/2.\ 1/2)^t$.

Clearly

$$W^t\gamma = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = J$$

The identification of the preceding equilibrium may be thought of as a first phase in a bargaining analysis process. These findings may be resumed as the description of the system in which players secure the amounts prescribed by the extended imputation x° provided the coalition that supports the corresponding payoffs forms.

Thus, before considering the formation of the grand coalition, the players may secure:

$(x_1°, x_2°, 0)$ if {1, 2} {3} form, since $x_1° + x_2° = v_{12}$

$(x_1°, 0, x_3°)$ if {1, 3} {2} form, since $x_1° + x_3° = v_{13}$

$(0, x_2°, x_3°)$ if {2, 3} {1} form, since $x_2° + x_3° = v_{23}$

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4 See vN-M(1947) p.243mnb.

5 The 3-person zero-sum game is the only case where x°, understood as one view of three different interrelated occurrences, is the solution.

III. SYNDICATE FORMATION PROCESS

When players realize the strategic-equilibrium that imbeds them, as members of a 2-person coalition they may become aware also, in considering the formation of the grand coalition, that they have the possibility of forming a *syndicate*. That is a coalition that behaves as a single player. By doing so, the characteristic function value of the game to each coalition may be used as a disagreement base. Then, a pure bargaining game emerges between the 2-player coalition that forms a syndicate v.s. the excluded one. We denote here by [i, j] the syndicate that forms versus player [k].

For each possible syndicate the new game that emerges is a two-player pure bargaining game that has the following characteristic function:

$\hat{u}([i, j]) = v_{ij}$, $\hat{u}([k]) = v_k = 0$, $\hat{u}([i, j], [k]) = v(N)$.

$$i \neq j \neq k,\ i, j, k = 1, 2, 3.$$

The corresponding 0-normalized game is:

$u([i, j]) = 0$, $u([k]) = 0$, $u([i, j], [k]) = v(N) - v_{ij} = e°_k$

$$i \neq j \neq k,\ i\ j\ k = 1, 2, 3$$

That is, if a syndicate [i, j] forms they divide the proceeds of the coalition v_{ij} obtaining each its maximum sustainable claim $x°_i$, and $x°_j$ respectively. Then, the syndicate may proceed to bargain with the excluded player k for the amount $e°_k$.

The solution to the emergent game can be summarized as follows;

$$y_{[i, j]} = \alpha_k\, e°_k,\ y_{[k]} = (1-\alpha_k)\, e°_k,$$

$$0 \leq \alpha_k \leq 1,\ i, j, k = 1, 2, 3,\ i \neq j \neq k$$

So that a general description of possible outcomes in the triangular 3-person cooperative game when the grand coalition forms preceded by possible formation of syndicates would be given by the following interrelated conditional system of imputations:

$(x°_1 + \beta_3\, \alpha_3\, e_3,\ x°_2 + (1 - \beta_3)\alpha_3\, e_3,\ (1 - \alpha_3)\, e_3)$ if $N = \{[1, 2], [3]\}$

$(x°_1 + \beta_2\, \alpha_2\, e_2,\ (1 - \alpha_2)e_2,\ x°_3 + (1 - \beta_2)\, \alpha_2\, e_2)$ if $N = \{[1, 3], [2]\}$

$((1 - \alpha_1)e_1,\ x°_2 + \beta_1\, \alpha_1\, e_1,\ x°_3 + (1 - \beta_1)\, \alpha_1\, e_1,)$ if $N = \{[2, 3], [1]\}$

The proposed general description implies a two stage formation process where the final outcome depends on how the grand coalition comes out to be. There are clearly three ways to arrived to the grand coalition: one for each possible syndicate.

Here, the power of a syndicate can be fully appreciated. The relative strength of a player in the two person coalitions appears to be dissolved when a syndicate forms against him.

The amount e_k is the incremental contribution of player k to the grand coalition N. This amount turns out to be the object of the negotiation between player [k] and the syndicate [i, j], given that only through the cooperation of these two agents, the amount e_k can be obtained.

To formalize the above constructive approach describing the possible outcomes that may occur as a consequence of the three players bargaining interaction, let C = {C_1, C_2, C_3} be the collection of two players subsets of N={1, 2, 3}, where C_1 = {2, 3}, C_2 = {1, 3} and C_3 = {1, 2}.

The corresponding characteristic row vectors are;

$W_1 = (0\ 1\ 1), W_2 = (1\ 0\ 1)\ W_3 = (1\ 1\ 0)$

Also, let the characteristic function values for the two person coalitions be

w1 = υ23, w2 = υ13 and w3 = υ12, with

$W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is the characteristic matrix for the collection C

The corresponding inverse matrix is:

$$W = \begin{bmatrix} -111 \\ 1-1 \\ 11-1 \end{bmatrix}$$

and the vector of characteristic values is

$$W = \begin{bmatrix} v_{23} \\ v_{13} \\ v_{12} \end{bmatrix}$$

Then,

$$x^0 = \begin{bmatrix} x^0_1 \\ x^0_2 \\ x^0_3 \end{bmatrix} = W^{-1}w = \begin{bmatrix} \frac{-v_{23}+v_{13}+v_{12}}{2} \\ \frac{v_{23}-v_{13}+v_{12}}{2} \\ \frac{v_{23}-v_{13}-v_{12}}{2} \end{bmatrix}$$

Given $v_{123} \geq v_{12} \geq v_{13} \geq v_{23}$, $x^0 \geq 0$ if $v_{12} \geq v_{13} + v_{23}$. That is, as long as the game is triangular there exists always a non-negative extended imputation x° of maximum sustainable claims and an interrelated system given by:

$$x^0 = \begin{bmatrix} 0 & x^0_2 & x^0_3 \\ x^0_1 & 0 & x^0_3 \\ x^0_1 & x^0_2 & 0 \end{bmatrix}$$

We define equivalently the system matrix X^o and the pair (x^o, C) so that X^o (x^o, C) and refer to either as the *strategic equilibrium* for the game. The conditionality of the system has the following interpretation: the amount v_{ij} is could be divided in $x°_i$ and $x°_j$ whenever coalition {i, j} forms to secure players i and j its maximum sustainable claims, i, j = 1, 2, 3, i $\neq$ j,

$$\left\{ \begin{array}{lccccc} & x_1^0 & x_2^0 & x_3^0 & \textit{conditional} & S \\ v_{23} = & 0 & 1 & 1 & \textit{if} & \{2,3\} \\ v_{13} = & 1 & 0 & 1 & \textit{if} & \{1,3\} \\ v_{23} = & 1 & 1 & 0 & \textit{if} & \{1,2\} \end{array} \right\}$$

The players could secure their corresponding maximum claims $x°_1$, $x°_2$ and $x°_3$ as attainable bargaining alternatives in two out of three possible outcomes of the game when cooperation takes place.

The value added, marginal contribution or incremental value $e°_i$ of player i, to the grand coalition value, i = 1, 2, 3, is given by:

$e°_1 = v_N - v_{23}$, $e°_2 = v_N - v_{13}$, $e°_3 = v_N - v_{12}$

Always $e°_i \geq 0$, whenever v is supper-additive: $v_N - v_S \geq 0$, $S \subseteq N$

Here we make a distinction between the formation of a coalition and the one of a syndicate. A coalition is a potential alliance that does not preclude the player to consider and negotiate the formation of other alliances. A syndicate is a formal structure that cannot be dissolved once it forms. It becomes a single unit that *de facto* invalidates all other coalitions where the players in the syndicate might belong in the coalition formation process. Players are assumed to lose the unilateral decision making implicit right to negotiate with players outside the syndicate. For any practical purpose, players in a syndicate have no identity at all.

We may reasonably conjecture that before a syndicate forms the corresponding coalition must form. For a coalition to form, a formal agreement on how to divide the proceeds of the coalition must take place. We may expect that the players will not object to dividing the value of the coalition so that each player secures his maximum sustainable claim. Then, the amount to be received by each player should be precisely the claim we know is prescribed by the extended imputation x°. After all, x° summarizes the fundamental equilibrium of the game. The payoffs in x° may become, using --- Harsanyi´s terms, the agreed "disagreement dividends" for the players members of the syndicate.

A second agreement must take place before a syndicate forms. If the players are to act as a single interest, they better agree in advance on how to split the amount to be received when the bargaining over the added value of the excluded player is settled. Hence, if syndicate v.s. [k] forms, becomes a *disagreement value* and clearly $x°_i$ and $x°_j$ *disagreement payoffs* The part of v_N to be negotiated with the excluded player k is and the general solution to this bargaining game is well known to be:

$$y^0_{[i,j]} = \alpha_k e°_k$$

$$y^0_{[k]} = (1-\alpha_k)\, e°_k \; 0 < \alpha_k < 1$$[6]

The necessary agreement between players i and j must be on how the amount $\alpha_k\, e°_k$ obtained by syndicate [i, j], in bargaining with player k, is to be divided between them. So, the agreement must specify a pair of complementary proportions, say β_k for player i and $(1 - \beta_k)$ for player j so that player i will obtain $\beta_k(\alpha_k\, e°_k)$ and player j receives $(1 - \beta_k)\, (\alpha_k\, e°_k)$, where $0 < \beta_k < 1$.

Thus, when considering the formation of the grand coalition and syndicates are allowed to form by the rules of the game, our original conditional systemic equilibrium evolves into the conditional system:

$$0 < \alpha_i < 1,\ 0 < \beta_i < 1,\ i = 1, 2, 3$$

This general description of possible outcomes may be viewed as an emergent process composed of two phases. I may be obtained as the sum of the two matrices that correspond to two interdependent conditional systems: (1) A conditional system that summarizes the maximum sustainable claims that result from the negotiations among players as potential members of coalitions; coalitions that are exhibited as bargaining alternatives to less desirable outcomes, and (2) A conditional system that summarizes the internal and external agreements of the syndicates.

[6] The open interval for α indicates that some positive amount must be received by the players if they are to cooperate or agree join in a coalition.

Phase I[7] Phase II

$$\begin{bmatrix} 0 & x^0_2 & x^0_3 \\ x^0_1 & 0 & x^0_3 \\ x^0_1 & x^0_2 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_1 e^0_1 & \beta_1(1-\alpha_1)e^0_1 & (1-\beta_1)(1-\alpha_1)e^0_1 \\ \beta_2(1-\alpha_2)e^0_2 & \alpha_2 e^0_2 & (1-\beta_2)(1-\alpha_2)e^0_2 \\ \beta_3(1-\alpha_3)e^0_3 & (1-\beta_3)(1-\alpha_3)e^0_3 & \alpha_3 e^0_3 \end{bmatrix}$$

$$0 < \alpha_I < 1,\ 0 < \beta_I < 1,\ i=1, 2, 3$$

The departing point for syndicate bargaining is the identification of disagreement payoffs given by the strategic equilibrium extended imputation x°.

At this point if the grand coalition is to form, players in a potential syndicate may consider securing the equilibrium claims of their coalition, leaving the payoff to third player to be its incremental contribution. They may also decide to dispute the added value with the excluded player by forming a syndicate that requires the specification of the internal and external division rates.

In any case our strategic equilibrium system induces a syndicate bargaining departure stage, namely the specific outcomes of the general description of possible outcomes where the α´s equal 1 and the β´s equal 0. This systemic solution stage is given by the conditional system:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} e^0_1 & x^0_2 & x^0_3 \\ x^0_1 & e^0_2 & x^0_3 \\ x^0_1 & x^0_2 & e^0_3 \end{bmatrix} \text{if} \begin{bmatrix} \{2,3\} \\ \{1,3\} \\ \{1,2\} \end{bmatrix}$$

The reader may readily verify that for $\Delta = v_N - x°(N)$ and $J_i = (\delta_{ij})_{3x1}$ then the syndicate bargaining departure points x, y , z are given by

$$x = x° + \Delta J_1,$$

$$y = x° + \Delta J_2 \text{ and}$$

$$z = x° + \Delta J_{3,}$$

A. Strategic-Equilibrium Based Possible Outcomes

In case our analysis has exhausted all strategic possibilities open to the players (which may not be the case), we may conjecture this emergent conditional system as the general description of possible outcomes to the 3-person general sum game. If other less intuitive strategic consi-

[7] We have change the order in which the coalitions are listed in relation to our first approach above for notational convenience.

derations have been omitted, we may retake them later.

Here clearly indicate the proportion that corresponds to player i, of his marginal contribution to when he negotiates with syndicate [j, k]. The proportions and (1-)´s indicate the complementary splitting proportions of the syndicate's gain between the two members. These must be defined by an agreement previous to the syndicate formation.

Clearly, the above cooperation rates emerge as necessary agreements when the grand coalition forms through the previous formation of a syndicate.These ´s and ´s together with the extended imputation x° are the essential determinants of the possible outcomes of the game, so we define:

$\delta°_i$: *Coalition´s strategic-equilibrium cooperation rate,* $\delta°_i = x°_i /(x°_i + x°_i)$ so that $x°_i = \delta°_i\, v_{ij}$, $i \neq j$, i, j = 1, 2, 3,

β_i: *syndicate´s internal-agreement rate*, which establishes the way to split the future possible but undetermined gains of pure bargaining.

α_i: *syndicate´ external-agreement cooperation rate* for dividing the contribution $e°_i$ of player i, to form the grand coalition with syndicate [j, k].

It is possible to have β_i as a monotonic non linear function of α_i and the possible outcomes of syndicate bargaining wouldn't be the linear continuum that takes place when β_i is chosen independently on α_i.

Now we proceed to look at our general description above in three clearly distinguishable cases; and in each case we consider two possible scenarios: (1) Syndicates are forbidden by the rules of the game and (2) Syndicates may form if the potential members so desire and agree upon.

Case I ($v_N = x°(N)$). In this case the core of the game exists and consists of a singleton that is $C(\Gamma) = \{x^0\}$, also we can readily show that $v_N = x°(N)$ implies $x°_i = e°_i,\ x^0 = (e°_1, e°_2, e°_3)$.

Our conditional system above becomes:

$$\begin{bmatrix} \alpha_1 e^{\circ}_1 & e^{\circ}_2 + \beta_1\left(1-\alpha_1\right)e^{\circ}_1 & e^{\circ}_3 + \left(1-\beta_1\right)\left(1-\alpha_1\right)e^{\circ}_1 \\ e^{\circ}_1 + \beta_2\left(1-\alpha_2\right)e^{\circ}_2 & \alpha_2 e^{\circ}_2 & e^{\circ}_3 + \left(1-\beta_2\right)\left(1-\alpha\right)^{\circ} \\ e^{\circ}_1 + \beta_3\left(1-\alpha\right)^{\circ} & e^{\circ}_2 + \left(1-\beta_3\right)\left(1-\alpha\right)^{\circ} & \alpha^{\circ} \end{bmatrix}$$

$$0 < \alpha_I < 1,\ 0 < \beta_I < 1,\ i=1, 2, 3$$

This solution system, graphed as interrelated separate occurrences in the imputation simplex viewed as a two dimensional object gives the following figure:

The graph reflects the order in which the parameters must be agreed upon in the syndicate inner and outer negotiations. First the syndicate internal agreement of its members must be ratified as a rational prerequisite to its formation. This appears as an obvious rational step, to avoid possible future conflict between the members. Then the external rate α is negotiated in a pure bargaining game.

However, if the members of the syndicate agree to settle the complementary division rate β, say a posteriori, it becomes clear that in case of disagreement, either a third party must enter to arbitrate the division of the syndicate´s gain or the members of the syndicate may find themselves in what we may term a "prelude to serious conflict" that may end up in players taking measures that are not contractually self enforcing.

In such case the graph of the solutions would be as shown in the following figure:

We may readily verify that the first of the two solutions obtained is a vN-M solution while the second one is not.

Figure 3.1 Strategic-equilibrium general solution when vN = x°(N) and syndicates negotiate α with excluded player after its members have agreed on β.

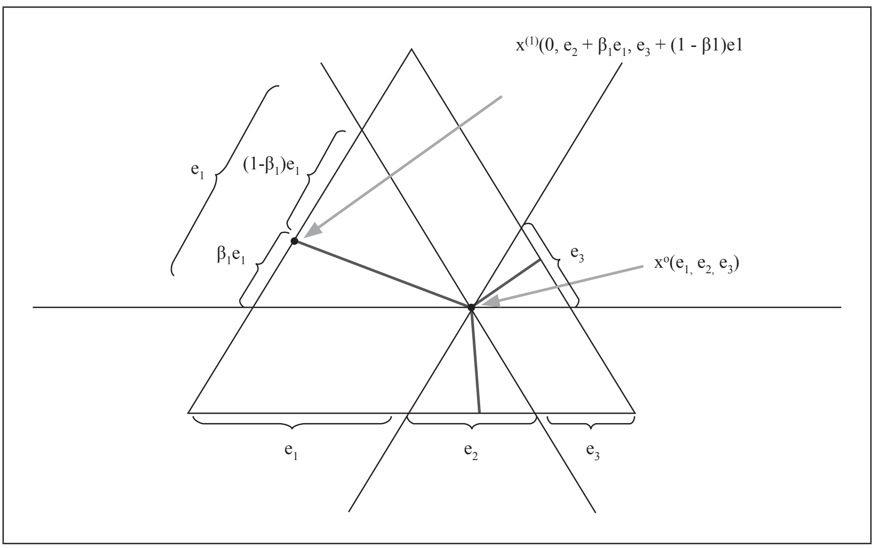
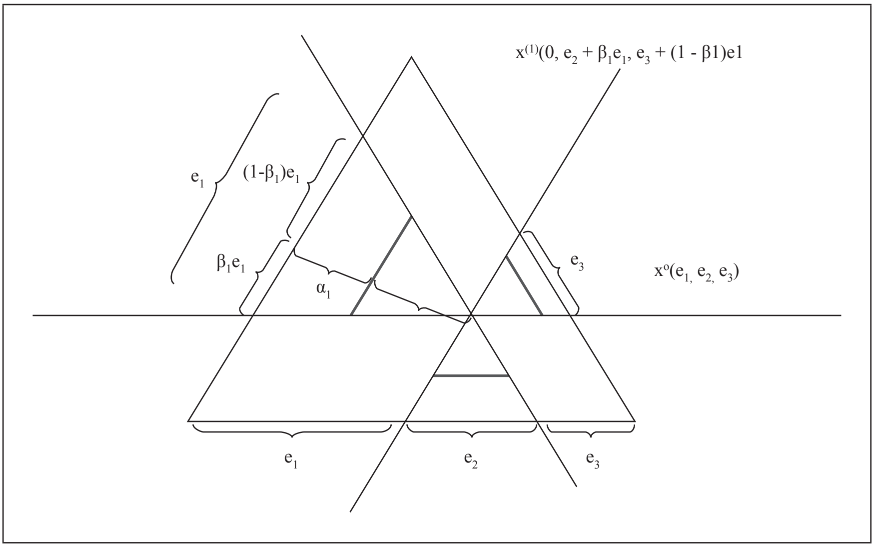


Figure 3.2 Strategic-equilibrium solution when $v_N = x°(N)$, α is reach then β is negotiated internally



Remark: It is remarkable that of the two particular strategic-stability possible outcomes that emerge as consequences of having a different sequential order for the same agreement decisions; the one that leads to possible uncontrolled conflict is precisely the one that is not a vN-M non-discriminatory solution. It is even more remarkable the vN-M solutions show to be sensitive to those sources of instability.

Case II $x°(N) > v(N)$ ($\Delta < 0$)

Whenever the value level x(N) of an imputation x is less than the strategic-equilibrium value level of the game x°(N), the core is empty, $C(\Gamma) = \phi$ whenever $x°(N) > x(N) = v(N)$. That is, if the characteristic function value to the grand coalition is less than the strategic-equilibrium value level of the game the core is empty. No imputation can satisfy all coalitional rationality conditions since the value-level of any imputation is v(N) and according to proposition 2.2, for an extended imputation to satisfy all these conditions, its value level must at least equal to the strategic–equilibrium value-level of the game x°(N). The graph of our strategic-equilibrium solution:

$$0 < \alpha_i < 1, 0 < \beta_i < 1, i=1, 2, 3$$

For given β (syndicate internal agreement rate), the graph of the solutions for variable division rate of the negotiated incremental contribution of the excluded players the graph of the solution is given below:

All the observations made in Case I regarding the sequential order in which the parameters α and β are agreed, determine equally whether the solutions obtained constitute or not vN-M solutions.

If the syndicates form without defining β (the way they will split the proceeds of the bargaining for the incremental contribution of the excluded player) and they settle for an α with the excluded player the graph with α fixed and β variable would look as in the following graph:

Again in this case by considering a vN-M non discriminatory solution as the one in figure 3,3 the weaker players in the coalition bargaining for the maximum sustainable claims are the one that would benefit more in forming a syndicate against player 1 for the contribution of such player is the largest and hence there is more to bargain for. Here again we run into the *stronger player paradox*. In our constructive analysis, it is clear that of the three interdependent solution branches one dominates the other two and our final selection as solution would be the dominant branch.

The resulting solution is not a vN-M solution unless as in Bayes theorem for conditional probability a new outcome space is re-defined with the occurrence of the conditioning event.

Figure 3.3 A Strategic-equilibrium solution when Δ <0. Also a vN-M solution

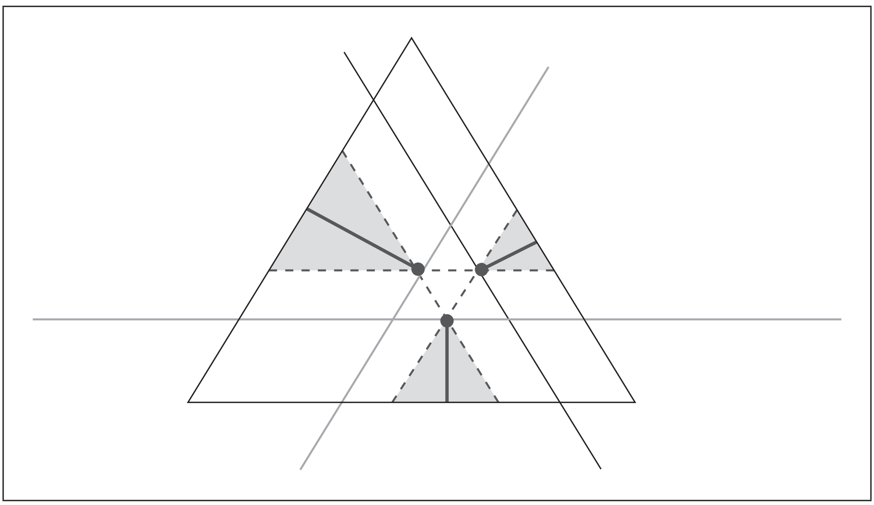


Figure 3.4 Strategic-equilibrium solution when Δ <0. Not a vN-M solution

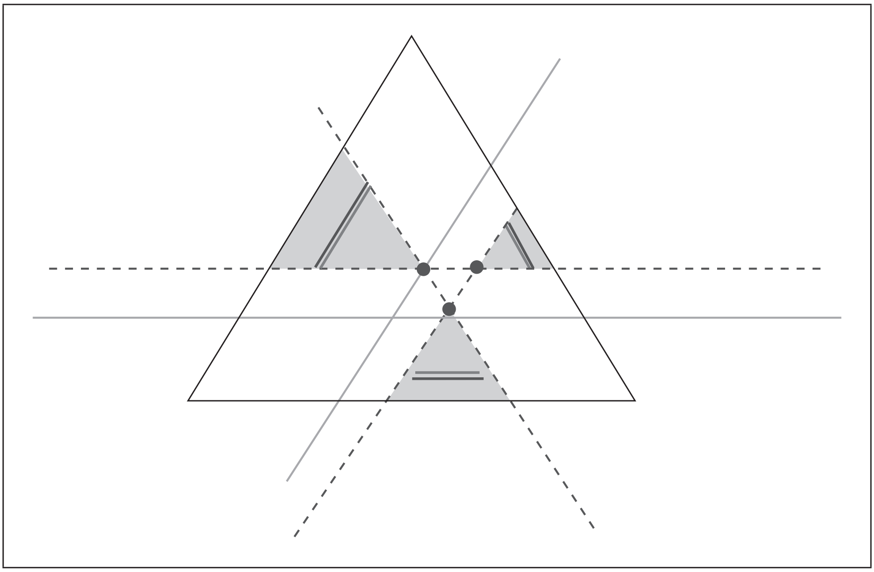
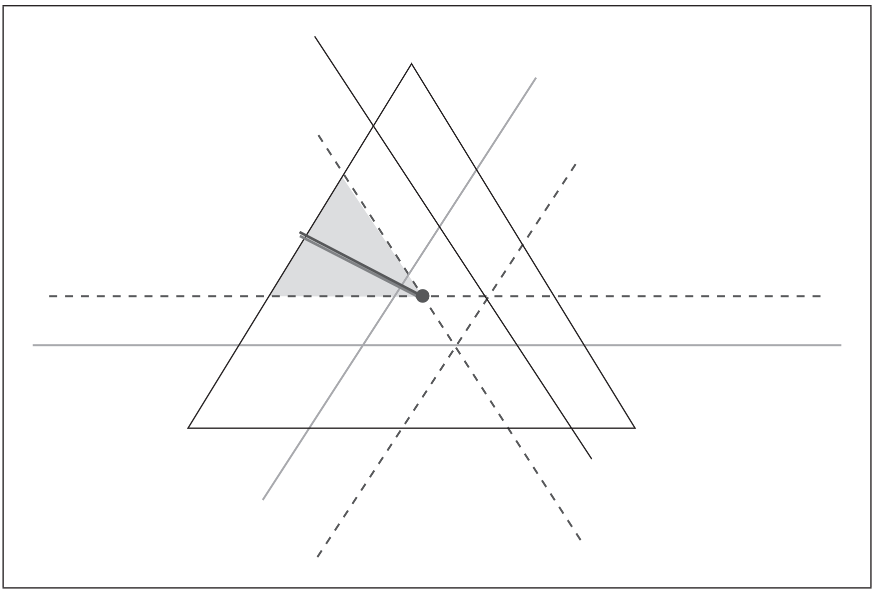


Figure 3.5 Any non-discriminatory vN-M solution where there is a stronger player defeats itself into one undisputable possibility against the stronger player



IV. DISCRIMINATORY SOLUTIONS AND THE STRONGER PLAYER PARADOX

Our constructive approach within a systems perspective based on the strategic equilibrium concept allowed us to uncover new dimensions of game theoretical analysis. Of special importance is the identification of several stages of discontinuous levels of rationality that emerge as the players expand their level of consciousness on the possible bargaining developments and the strategic alternatives at their disposal.

We have seen that if syndicates are allowed to form by the rules off the game, and if players are not symmetric there will always be a stronger player and such player will end up being the looser. The weaker players invariably will gang up against the stronger player when confronted with such solution. Such condition is endemic to all von Neumann and Morgenstern non-discriminatory solutions for the three person general sum cooperative game. If we give restrictive interpretations to vN-M solutions, we are bound to qualify vN-M discriminatory solutions as self contradictory except in the symmetric players case. So we proceed not to make judgments and rather to keep open the scope of possibilities.

Continuing with our constructive approach to possible outcomes of rational interacting behavior among the players, we are at a stage were non-discriminatory vN-M solutions have emerge provided there is an ordered sequence of binding agreements. Those solutions exhibit three conditional interrelated possibilities with one of them clearly superior, for a decisive number of players, than the other two.

So in view of such inevitable outcome for player 1 the stronger in the sense that

$$s_1 = \max_{j\in N} s_j \text{ where } s_j = \frac{1}{|\;|}\sum_{\in} (\quad)$$

The obvious question is: how[8] can I, player 1, make binding agreements to lower my profile, eliminate the attractiveness of my condition and, if possible, to secure my inclusion in the grand coalition with a payoff that does not depend on a two-player pure bargaining game.

[8] In "real life" we may observe myriads of strategies that players use to not to show a high profile when this profile is really high. These range from transfers of a utility like money through non-profit organizations to the actual constructions and modifications of perceived "realities" in stakeholders: players or potential players

Clearly, player 1 may voluntarily lower his strategic equilibrium conditional payoff $x°_1$ by increasing both players 2 and 3 equilibrium claims with a utility transfer ε of his $x°_1$ claim. This conditional transfers may be made mainly to avoid being gang-up against which presumably is bound to occur if no bargaining alternative is offered to players 2, and 3[9]. Player 1 may have to make binding agreements to that if the grand coalition does not form he will split the 2-person characteristic function value of the coalition accordingly: $x°_1 - \varepsilon, x°_h + \varepsilon$ h = 2, 3. If the grand coalition forms players 2 and 3 will guarantee the same amount to player, unless syndicates form. In such case the bargaining for the players incremental contribution $e°_j$, develops with respect to the displace equilibrium induced by player 1 voluntary cession of his sustainable claim.

The size of the utility transfer must be sufficient to reduce $e°_1$ at least to the size of the second largest marginal contribution so that $2\varepsilon \geq e°_1 - e°_2$ with this strategy player 1 has no longer the most attractive resource to be bargained against by the two syndicated players.

The utility transfer must be an admissible one in the rational field of extended imputations that may be supported simultaneously by coalitions {1, 2} and {1, 3} contrary to the natural direction of transfers in such cover structure of N: $\xi = (-\varepsilon, \varepsilon, \varepsilon)^t$

Note that for $W^{(1)} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ the characteristic matrix of the cover collection C = {{1, 2}, {1, 3}}, the transfer fulfills the admissibility requirement $W^{(1)}\xi = 0$.

The binding agreements subscribed by player 1 makes possible the new strategic equilibrium. Basically player 1 promises player 2 that if {1, 2} forms, the distribution of v_{12} will be $(x°_1 - \varepsilon, x°_2 + \varepsilon, 0)$, $\varepsilon > 0$. Similar with player 3, if {1, 3} forms the payoffs will be $(x1° - \varepsilon, 0, x°_3 + \varepsilon)$, and clearly, since v23 cannot support the new claims ,player 1 must leave open the possibility for players 2 and 3 to use those new claims as disagreement payoffs if they try to form the grand coalition from his syndicate [2, 3] versus [1]. In this case, the bargaining alternative is the grand coalition and and the modified bargaining departure point for syndicate [2, 3]. The corresponding equilibrium payoff is $(e°_1 - 2\varepsilon, x°_2 + \varepsilon, x°_3 + \varepsilon)$.

It evolves out of a non discriminatory vN-M solution $e°_1 - 2\varepsilon < e°_2$ implies $\varepsilon > (e°_1 - e°_2) / 2$.

[9] The reasons given when the strong player paradox was introduce above.

Figure 4.1 Admissible self-induced penalty transfers to solve the stronger player paradox

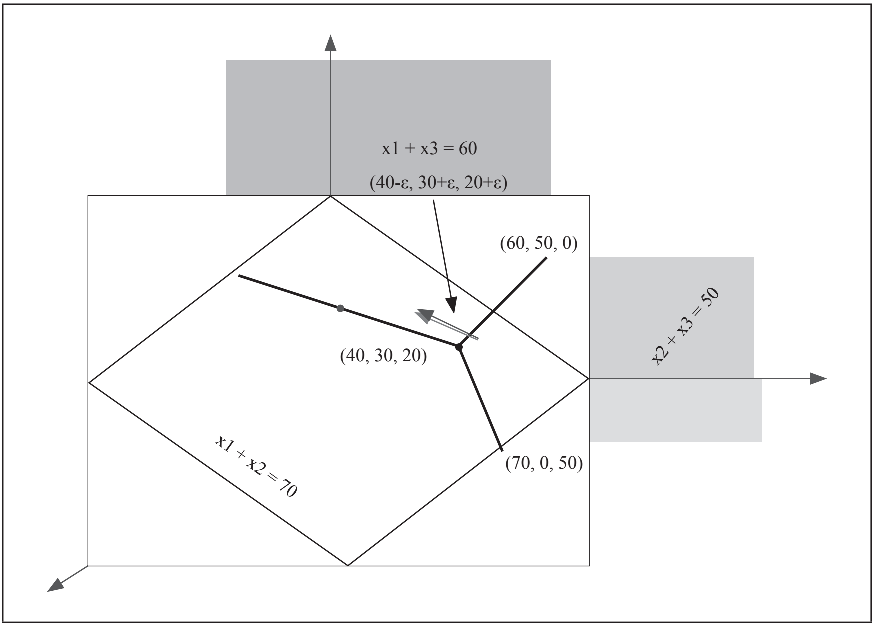
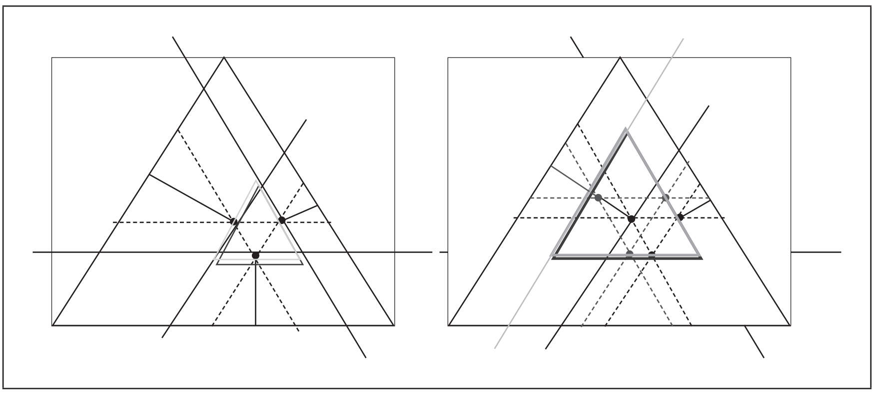


Figure 4.2 Emergence of a vN-M discriminatory solution as a strategic measure to counter the stronger player paradox



It becomes clear then, that the relative advantages of coalition bargaining generates a natural adverse reaction against the stronger players and that the possible remedy so far identified is to give up the privileges rightly deserved but with the seeds of trouble in subsequent stages of the development of the game. Such measure will force in many cases the second strongest player to take similar measures because after player 1 lowers his profile to avoid, player 2 may becomes the object of syndicated attacks.

We can foresee a dynamic development that necessarily ends up with a modified game and the scenario is one of equal opportunities for all players.

V. SYNDICATE POWER AND THE STRONGER PLAYER PARADOX

One may think that all strategic considerations have been examined. However, we may observe that of the three possible ways that 2-players coalitions may form, the one with the largest excess is the one that rational players would choose form. The reasoning may go as follows: We players 2 and 3 can secure the amounts $x°_2$ and $x°_3$ in two of the three cases. By getting together and forming a syndicate versus player 1, not only we secure with certainty our maximum sustainable claims but also as a syndicate we may dispute an additional amount, higher than in any other syndicate that under similar rate conditions, the share we obtain of the syndicates gain would be higher. We may consider players 2 and 3 forming a syndicate to bargain with player 1 for the amount $e°_1$., under similar conditions, say the syndicates internal agreement rate follows an egalitarian standard, so that $\beta = 1/2$ and the syndicates external bargaining rate α is determined by a democratic standard so that $\alpha = 2/3$ then

$$x^*_2([i,j]) = \begin{cases} x°_2 + (1/3)\, e°_3 \text{ if } [1, 2] \text{ forms} \\ 0 + (1/3)\, e°_2 \text{ if } [1, 3] \text{ forms} \\ x°_2 + (1/3)\, e°_1 \text{ if } [2, 3] \text{ forms} \end{cases}$$

Similarly

$$X^*_3([i, j]) = \begin{cases} 0 + (1/3)\, e°_2 \text{ if } [1,2] \text{ forms} \\ x°_3 + (1/3)\, e°_2 \text{ if } [1, 3] \text{ forms} \\ x°_3 + (1/3)\, e°_1 \text{ if } [2, 3] \text{ forms} \end{cases}$$

Since, v12 > v13 >v23 implies both $x°_1 > x°_2 > x°_3$ and $e°_1 > e°_2 > e°_3$, then x^*_2 and x^*_3 are both maximized in [2, 3], it follows that the player that can sustain the higher claim is the one who will end up ganged up against.

Paradoxically, the strongest player at the coalitional bargaining level becomes the weakest one at the syndicate bargaining level. This paradox appears here as an emergent characteristic that may be present in all games of cooperation and that can be readily recognized as a socioeconomic behavioral archetype.[10] The stronger player paradox requires a strategic response on the part of the excluded stronger players at the game theoretical level. This matter will be retaken later. Here our strategic-equilibrium solutions that are vN-M solutions too, suggests themselves not as actual solutions but as warning scenarios of the future: This is what unavoidably will happen unless somebody takes the appropriate strategic measures. This amounts to gain a deeper insight on the relative stability of vN-M discriminatory solutions.

The strategic-equilibrium solutions of the game with syndicates induce a bargaining process that appears to have "equal opportunity" for all as a fundamental systemic behavioral attractor. If the game is not regulated, leaving the players free to make decisions in the emerging bargaining processes, specifically free from rules that restrict coalitional behavior in forming syndicates, the players will run into the stronger player paradox. From there, comes the realization for the stronger players that only through voluntary modifications of the strategic-equilibrium for the game made as bargaining binding agreements, may the *stronger player paradox* be avoided.

The required modifications may take place as stronger players claim reduction agreements to increase the weaker players claims. That is, these are moves to equalize the bargaining power of the players leading invariably to an equal opportunity stage for all players. This emergent systemic meta-game attractor can be readily obtained by finding the strategic-equilibrium of the parametric partition function derived game that emerges with the syndicate's formation possibilities. This will be shortly demonstrated when the stronger-player paradox is re-exam after the following cases.

Case ($v_N > x_{(N)}$)Whenever the characteristic function value of the game to the grand coalition is strictly greater than the optimal strategic equilibrium level x°(N),[11] we know that the cardinality of the core is greater than 1, that is, the core $C(\Gamma) \neq Ø$ is not a singleton since, $| C(\Gamma)| > 1$. In this case the core has infinitely many

[10] This paradox has popular resonance and is warned of in conventional wisdom slogans and in even in religious maxima - : "the first will be the last " be "humble as lambs and wise as foxes" the humble ones will be exalted and exalted ones will be brought down" It also relates to popular recommendations such as "to keep a low profile".

[11] The term optimal equilibrium level refers to the fact that for n>3 there are many strategic equilibriums at different xd-imputation value levels, of those only the ones in the same value level and satisfy coalitional rationality conditions, constitute the fundamental equilibrium of the game.

solutions bounded by the coalitional rationality and the group rationality conditions of all its imputations. Guided by our stated leit- motif: “always to look for the non-discriminating solutions”, we proceed to establish the following proposition:

Example Let $v_N = 100$, $v_{12} = v_{13} = v_{23} = 50$, $v_1 = v_2 = v_3 = 0$. The strategic equilibrium (x°, C) for the given game is given by:

$x° = (25, 25, 25)^t$, C = {{1,2}, {1,3}, {2,3}} The incremental contribution of each player is:

$$e°_1 = e°_2 = e°_3 = 50$$

The syndicate bargaining departure points x, y, z are:

$x = (50, 25, 25)^t$, $y = (25, 50, 25)^t$, $z = (25, 25, 50)^t$

The extreme points of the core a, b, c are given by

$$a = -x + y + z = (0, 50, 50)^t$$

$$b = x - y + z = (50, 0, 50)^t$$

$$c = x + y - z = (50, 50, 0)^t$$

In Figure 2.16 below, the line segments joining the intersections of individual and coalitional rationality constraints with the equilibrium extended imputation x° constitute the *rational bargaining field*.

The structure of the admissible utility transfer ξ_j for each player j=1, 2, 3

$$\xi_1 = (25, -25, -25)^t$$

$$\xi_2 = (-25, 25, -25)^t$$

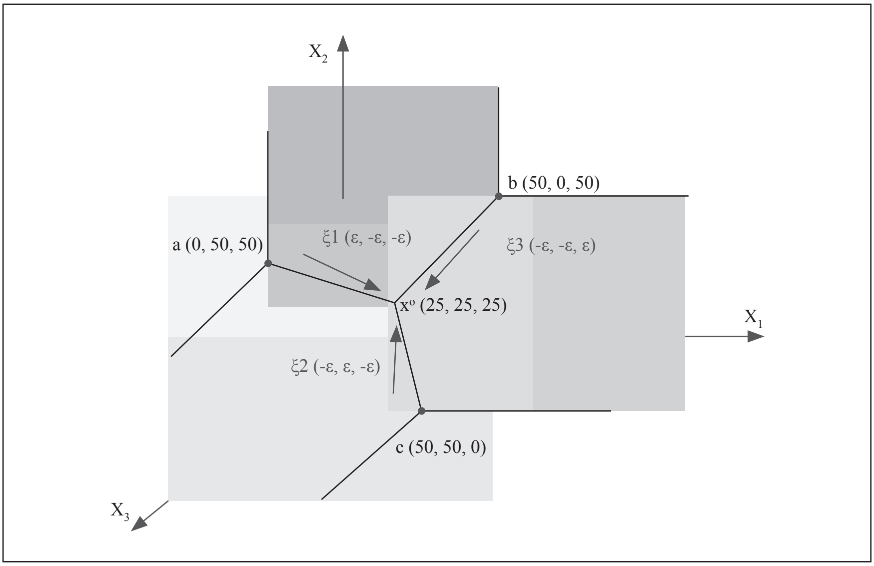
$$\xi_3 = (-25, -25, 25)^t$$

Utility transfers are indicated in figure 2.19 with arrows pointing toward x°. Player j makes use of his bargaining alternatives {i, j} and {j, k} to demand a utility transfer from players i and k. We observe that neither player i nor player k have a bargaining alternative to support a payoff claim x in the corresponding bargaining (open line) segment. Only at x° these players have an effective bargaining alternative to secure the corresponding claims. By forming coalition {i, k} they may obtain

The admissible utility transfer structure ξ_j for each player j = 1, 2, 3 is indicated with arrows pointing toward x°. Player j makes use of his bargaining alternatives {i, j} and {j, k} to demand a utility transfer from players i and k. We observe that that neither player i nor player k have a bargaining alternative to support a payoff claim in the corresponding bargaining segment. Only when x° is reached these players have the bargaining alternative {i, k}.

The underlying bargaining structure of the extended imputations in the

Figure 5.1 Utility transfers structure in ***rational field***[12]



rational field is the bargaining support for negotiations in the imputations ground, the induced transfer structure for player 1 is shown ion figure below:

Whenever $v_{13} + v_{23} < v_N$. the core no longer is enclosed by imputations not in the core. The s-core always maintain its triangular form se figure bellow:

As $v_{12} \rightarrow 0$, $C(\Gamma) \rightarrow I$ and the game becomes a Pure- Bargaining 3-person game.

Remark: in general, negotiating for core-outcomes is a completely different than negotiating in a game of pure bargaining where players have no bargaining alternatives and there are no syndicate formation possibilities.

Case II $x°(N) > v(N)$ $(\Delta < 0)$

Whenever the value level x(N) of an imputation x is less than the strategic-equilibrium value level of the game x°(N), the core is empty, $C(\Gamma) = \phi$ whenever $x°(N) > x(N) = v(N)$.

[12] The rational bargaining field consists of the extreme edges of the polyhedral set of all detached extended imputations. vN-M definition of detached extended imputations include all subsets of N. Here, only proper subsets are included.

Figure 5.2 Utility transfers structure in *imputation simplex*

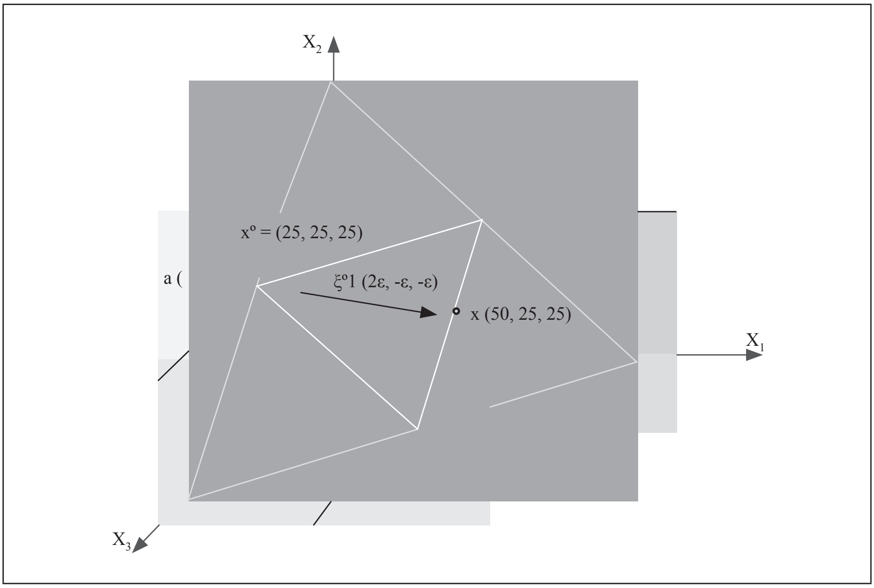
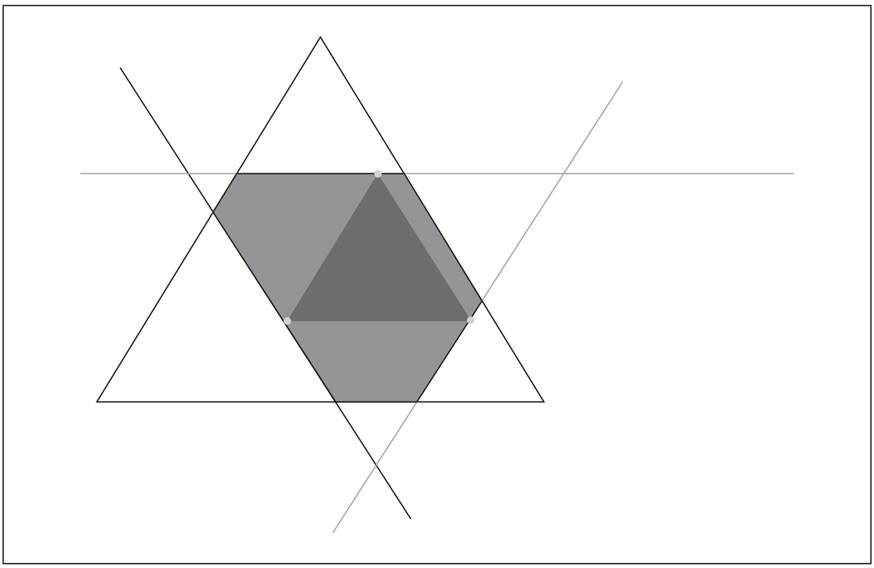


Figure 5.3 Core of the game when $v_{12} + v_{13} < v_N$



That is, if characteristic function value to the grand coalition is less than the strategic-equilibrium value level of the game the core is empty. No imputation can satisfy all coalitional rationality conditions since the value-level of any imputation is v(N) and according to proposition 2.2, for an extended imputation to satisfy all these conditions, its value level must at least equal to the strategic–equilibrium value-level of the game: x°(N).

The graph of our strategic-equilibrium solution:

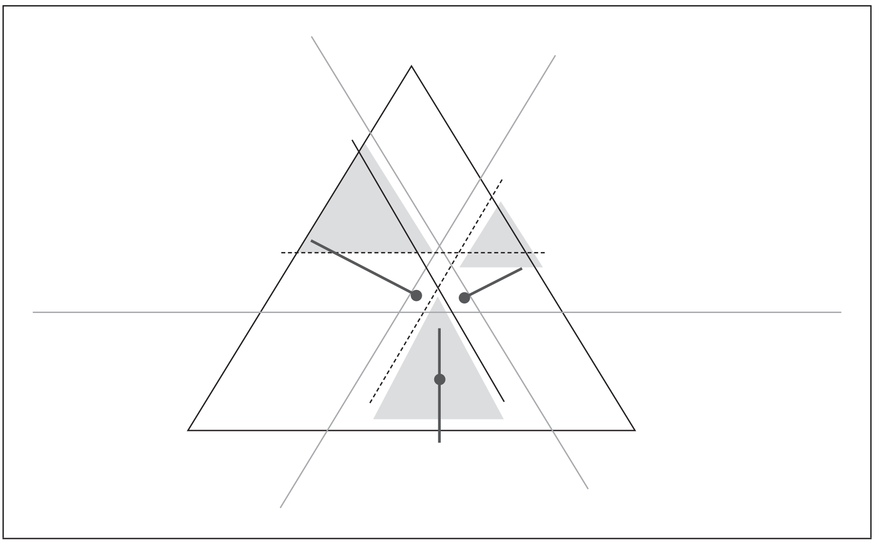
$$0 < \alpha_i < 1,\ 0 < \beta_i < 1,\ i = 1, 2, 3$$

For given β (syndicate internal agreement rate), the graph of the solutions for variable divisions of the negotiated incremental contribution of the exluded players the gtaph of the solution is given below:

All the observations made in Case I regarding the sequential order in which the parameters α and β are agreed determine equally whether the solutions obtained constitute or not vN-M solutions.

If the syndicates form without defining β (the way they will split the proceeds of the bargaining for the incremental contribution of the excluded player) and they settle for an α with the excluded player the graph with α fixed and β variable would look as the one that follows below:

Figure 5.4 Strategic-equilibrium solution when Δ < 0. Also a vN-M solution



Again, in this case by considering a vN-M non discriminatory solution as the one in figure 5.4 the weaker players at the coalitional formation level, bargaining for the maximum sustainable claims, are the ones that would benefit more in forming a syndicate against player 1 for the contribution of such player is the largest and hence there is more to bargain for. Here again we run into the *stronger player paradox*. In our constructive analysis, it is clear that of the three interdependent solution branches one dominates the other two and our final selection as solution would be the dominant branch.

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VI. VN-M DISCRIMINATORY SOLUTIONS AND THE STRONGER PLAYER PARADOX

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Figure 5.5 Strategic-equilibrium solution when Δ <0. Not a vN-M solution

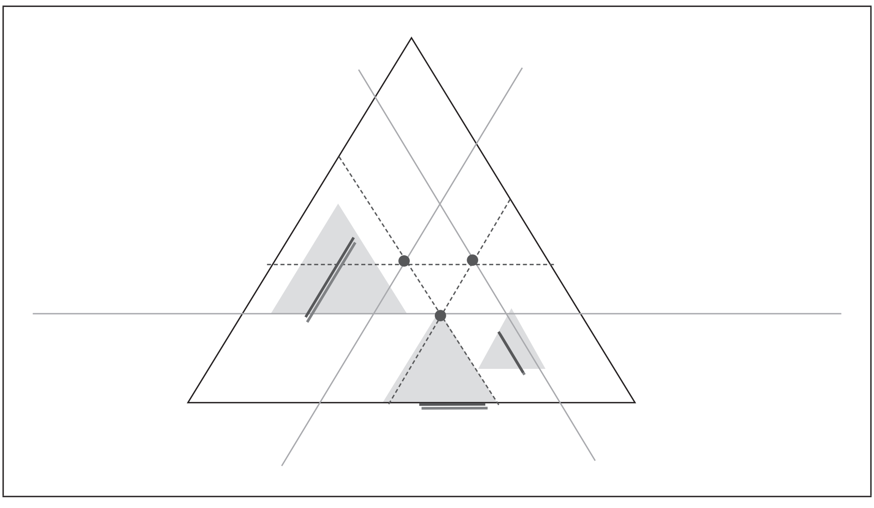
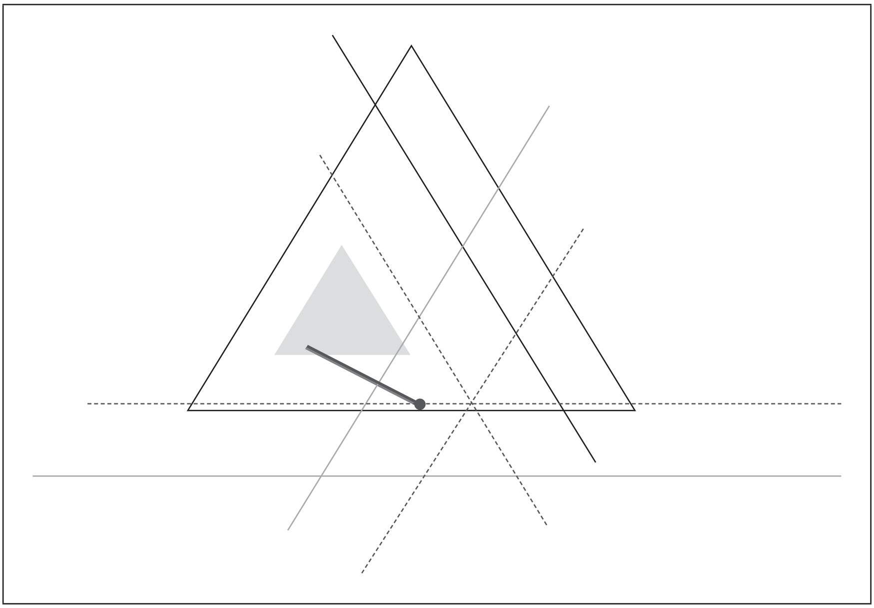


Figure 5.6 Any non-discriminatory vN-M solution where there is a stronger player defeats itself into one undisputable possibility against the stronger player.



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The obvious question is: how can I, player 1, make binding agreements to lower my profile, eliminate the attractiveness of my condition and if possible, to secure my inclusion in the grand coalition with a payoff that does not depend on a two-player pure bargaining game?

Clearly, player 1 may voluntarily lower his strategic equilibrium conditional payoff $x°_1$ by increasing both players 2 and 3 a utility transfer ε of his $x°_1$ claim, on condition that if the grand coalition does not form he will split the the 2-person characteristic function value of the coalition accordingly: $x°_1$ - ε, $x°_h$ + ε h =2, 3. If the grand coalition forms players 2 and 3 will guarantee the same amount to player, unless syndicates form. In such case the bargaining for the players incremental contribution $e°_j$, develops with respect to the displace equilibrium induced by player 1 voluntary cession of his sustainable claim.

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Note that for $W^{(1)}$ = the characteristic matrix of the cover collection C = {{1, 2}, {1, 3}}, the transfer fulfills the admissibility requirement $W^{(1)} \xi = 0$.

It becomes clear then, that the relative advantages of coalition bargaining generate a natural adverse reaction against the stronger players and that the possible remedy so far identified is to give up the privileges rightly deserved but with the seeds of trouble in subsequent stages of the development of the game. Such measure will force in many cases the second strongest player to take similar measures because after player 1 lowers his profile, player 2 may become the object of syndicated attacks.

We can foresee a dynamic development that necessarily ends up with a modified game and the scenario is one of equal opportunities for all players.

Figure 6.1 Admissible induced transfer to solve the stronger player paradox

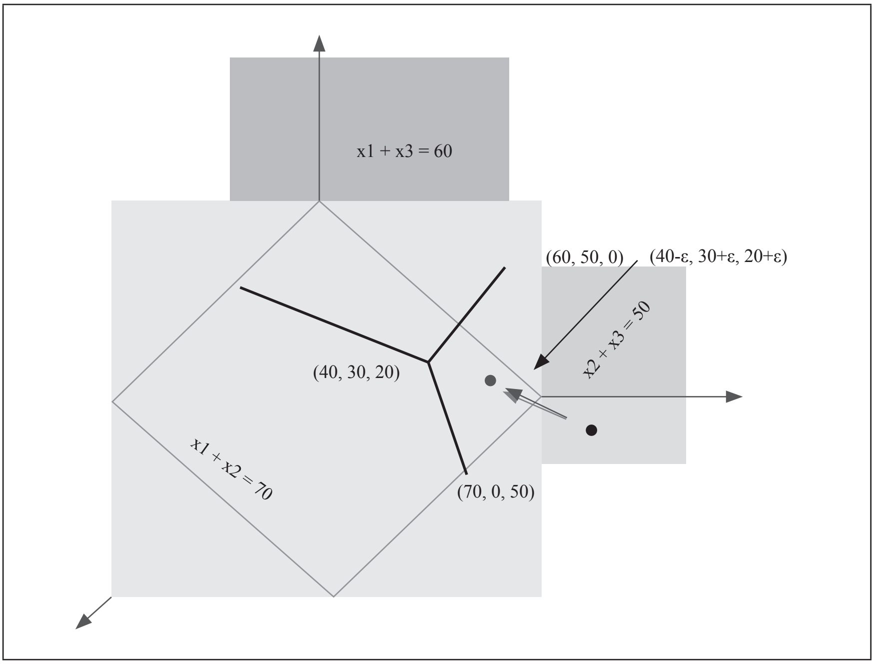
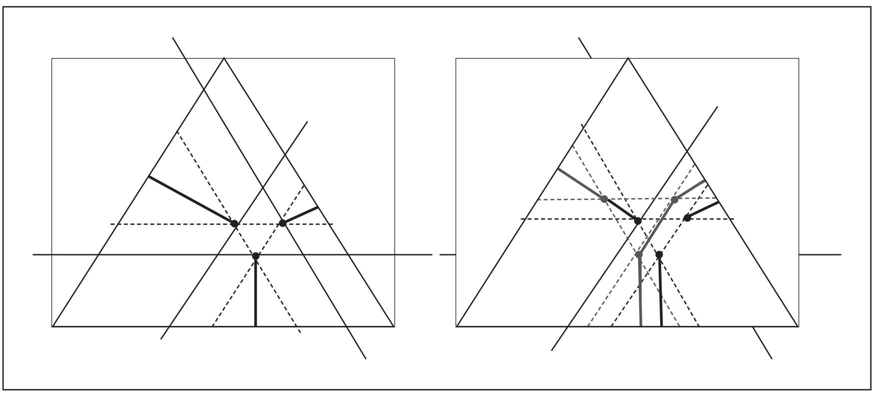


Figure 6.2 Emergence of a vN-M discriminatory solution as an strategic measure to counter the stronger player paradox. It evolves out of a non discriminatory vN-M solution



VII. EQUAL-OPPORTUNITY FOR ALL PLAYERS ATTRACTOR: THE META-STRATEGIC- EQUILIBRIUM OF A COOPERATIVE GAME?

When players have total freedom for choice and association and the only restrictions are those freely accepted in binding agreements, the formation of syndicates is a real possibility that has to be considered. If the dividends of the syndicates are taken as the prescribed by our strategic–equilibrium these become the disagreement payoffs of the syndicated players and our strategic equilibrium solutions emerge as logical consequences of rational player's interaction. Actually when considering the formation of syndicates, a derived game emerges. It natural formulation is in partition function form and it is always a constant sum game.

Thus, for the triangular 0-normalized 3-person cooperative game with transferable utility, and characteristic function values v_{N}, v_{12}, v_{13} and v_{23}

The derived game has the following partition function form:

If syndicate:

[1] [2] [3] forms $u(\phi) = 0$

[1, 2] [3] forms $u'(\{1,2\}) = v_{12} + (1-\alpha_3)\, e°_3$, $u'(\{3\}) = \alpha_3\, e°_3$

[1, 3] [2] forms $u'(\{1,3\}) = v_{13} + (1-\alpha_2) e°_2$, $u'(\{2\}) = \alpha_2\, e°_2$

[2, 3] [1] forms $u'(\{2,3\}) = v_{12} + (1-\alpha_1) e°_3$, $u'(\{1\}) = \alpha_1\, e°_1$

If $[h, j] \cup [k] = N$ $u'(\{1, 2, 3\}) = v_N$
$0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$

The parameters α_j, j = 1, 2, 3 are respectively the syndicate external bargaining agreement rate .

The derived game u´in strategic equivalent 0-nornalized form becomes:

$u(\{1, 2\}) = v_{12} + (1 - \alpha_3) e°_3 - \alpha_1\, e°_1 - \alpha_2\, e°_2$, $u(\{3\}) = 0$

$u(\{ 1, 3\}) = v_{13} + (1- \alpha_2)\, e°_2 - \alpha_1\, e°_1 - \alpha_3\, e°_3$, $u(\{2\}) = 0$

$u(\{ 2, 3\}) = v_{12} + (1- \alpha_1)\, e°_3 - \alpha_2\, e°_2 - \alpha_3\, e°_3$, $u(\{1\}) = 0$

$u(\{1, 2, 3\} = v_N - \alpha_1\, e°_1 - \alpha_2\, e°_2 - \alpha_3\, e°_3$, $u(\phi) = 0$

Note

$u(\{h, k\}) = v_{hk} + e°_j - (\alpha_1\, e°_1 + \alpha_2\, e°_2 + \alpha_3\, e°_3)$

$= v_{hk} + e°_j - E = v_{hk} + v_N - v_{hk} – E$

or

$u(\{h, k\}) = v_N - E$, h, k = 1, 2 ,3 $h \neq k$, also $u(N) = v_N - E$

The strategic-equilibrium extended imputation y° for the derived game constant sum game is given by:

$$y°_1 = (v_N - E) / 2$$

$$y°_2 = (v_N - E) / 2$$

$$y°_3 = (v_N - E) / 2$$

A sufficient condition for a triangular game (ie: $y° \geq 0$) Is to have $C(\Gamma) \neq \phi$ equivalently $\Delta > 0$ or $v_N > E$.The strategic-equilibrium extended imputation y° for the derived game is given by:

$y°_j = (u_{ij} - u_{ik} + u_{jk})/_2$ i, j, k =1, 2, 3, $i \neq j \neq k$.

That is,

$$y°_j = (v_N - E) / 2$$

In terms of the strategic equilibrium extended imputation for the original game:

Since

$$y°_j = (v_N - E) / 2$$

$$= v_N - v_{ik} - ½E - ½\, v_N + v_{ik}$$

$$y°_j = x°_j + v_{ik} - ½(E - v_N)$$

Clearly $y°_j \geq x°_j$ iff $v_{ik} - ½(E - v_N) \geq 0$, for j = 1, 2, 3.

Iff $v_{2\,3} \geq ½(E - v_N)$ or $2\, v_{2\,3} + v_N \geq \alpha\, e(N)$.

For $\Delta = 0$

Example. Let $v_{12} = 80$, $v_{13} = 70$, $v_{23} = 50$ and $v_{123} = 100$, $v(S) = 0$ otherwise. For $\Delta = 0$, $e°_j = x°_j$, j = 1, 2, 3 and $e(N) = x(N) = v_N$

Then, e_1 =50, e2 = 30 e3 = 20 and x° = (50 30 20), $v_N = 100$,

Consider $0 < \alpha < 1$ to be the standard syndicate bargaining external rate. It sets the proportion corresponding to the excluded player to the same for all syndicates, then $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$

For $u_{12} = u_{13} = u_{23}$ =100- $\alpha(50 + 30 + 20)$, $u_{ij} \geq 0$ if $1 \geq \alpha$. The equilibrium y° is given by

In a 3-person pure-bargaining game, $v_N = 1$ and $v(S) = 0$ for all $S \subset N$.

$u_{ij} = v_N – E$ and $e°_j = v_N = 1$

$u_{ij} \geq 0$ if $v_N \geq \alpha\, e(N)$ or if $1 \geq \alpha 3$, or if $1/3 \geq \alpha$.

The strategic-equilibrium based outcomes of the game with syndicates took us to vN-M non-discriminatory solutions where a paradox emerges. The responses necessary to deal with the paradox induce a third bargaining stage. This stage is not characterized as the preceding stages by group interaction but by individual measures in a process that appears to converge to an "equal opportunity" for all as a fundamental systemic behavioral attractor.

If the cooperation among players is not regulated, leaving the players free from any force- inducement to make decisions other than maximization of derived and the enforcement of willing binding agreements, and more specifically, free from rules that restrict coalitional behavior in forming syndicates, the players will run into the stronger player-paradox. From there, comes the realization for the stronger players that only through voluntary modifications of the strategic-equilibrium for the game made as bilateral bargaining binding agreements, the consequences of the stronger player-paradox may be avoided. The required modifications may take place as stronger players claim reduction agreements to increase the weaker player's claims. This particular strategic response gives place to the emergence of self-induced discriminatory vN-M solutions. That is these are moves to equalize the bargaining power of the players. These moves lead invariably to an equal opportunity stage for all players. This emergent systemic meta-game attractor can be readily obtained by finding the strategic-equilibrium of the parametric partition function derived game that emerges with the syndicate's formation possibilities.

VIII. CONCLUDING REMARKS

The overall strategic-equilibrium based results introduces us to new dimensions of analysis in the theory of cooperative games and clearly allows to realize the importance of the strategic-equilibrium concept and the relevance of vN-M solutions for economic analysis. Nonetheless, it urges us to view the n-person game as a dynamic sequence of bargaining stages, to rethink the solutions of the n-person cooperative game and to reconsider the meaning of vN-M non-discriminatory stable sets as solutions, given that a-posteriori strategic responses of the players will limit the possible occurrences (solution outcomes) to the dominating branch of the given solution.

The results related to the strategic response to the stronger player paradox have far reaching implications that may forces us to rethink our conventional interpretations of economic processes. These have been presented here by contrasting a strategic solution to the stronger player paradox with the equal opportunity for all meta-game attractor. The implications for current approaches to economic behavior analysis appear intuitively far reaching and significant.

Recursive operations in a close system result in fixed points, attractors or eigen-behaviors. Clearly, cooperation and conflict may be viewed as iterative operations. Since the world economy is an operationally closed system, the e*qual opportunity for all attractor* may well be a major eigen- behavior that characterizes cooperation in free economies. The stronger player paradox allows us to see clearly why syndicates and cartels among other collusion type of cooperative behavior tend to be forbidden. Hence coactive measures evolved to get, to protect and improved the status-quo of the stronger players. These range from wars and economic takeovers to patient lobby and legislative processes that may make illegal some forms of cooperation, create norms and standards or as barrier entries, etc. The equal opportunity for all attractor is something that creative entrepreneurs, successful businessmen may want to subscribe at the policy level but at the bargaining level the strategies followed create at any cost the conditions to destabilized and block the naturally convergent cooperative processes. The above considerations gives us some reasoned hope that we may understand some day, how can be possible that "the peaceful ones will inherit the earth"?

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