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Confidence sets for asset correlations in portfolio credit risk

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Abstract

Asset correlations are of critical importance in quantifying portfolio credit risk and economic capital in financial institutions. Estimation of asset correlation with rating transition data has focused on the point estimation of the correlation without giving any consideration to the uncertainty around these point estimates. In this article we use Bayesian methods to estimate a dynamic factor model for default risk using rating data (McNeil et al., 2005; McNeil and Wendin, 2007). Bayesian methods allow us to formally incorporate human judgement in the estimation of asset correlation, through the prior distribution and fully characterize a confidence set for the correlations. Results indicate: i) a two factor model rather than the one factor model, as proposed by the Basel II framework, better represents the historical default data. ii) importance of unobserved factors in this type of models is reinforced and point out that the levels of the implied asset correlations critically depend on the latent state variable used to capture the dynamics of default, as well as other assumptions on the statistical model. iii) the posterior distributions of the asset correlations show that the Basel recommended bounds, for this parameter, undermine the level of systemic risk.

JEL Classification: G32, G33, C01.
Keywords: Asset correlation, non-Gaussian state space models, Bayesian estimation techniques, zero-inflated binomial models.

Conjuntos de confianza para la correlación de activos en el riesgo de crédito de un portafolio

Resumen

Las correlaciones entre los activos de un portafolio crediticio, son parámetros de suma importancia para la estimación del riesgo crediticio y capital económico de una institución financiera. La literatura especializada en la estimación de las correlaciones entre los activos, que utiliza información de migraciones entre las calificaciones de riesgo, se ha concentrado principalmente en la estimación puntual de los parámetros, desconociendo la incertidumbre alrededor del estimador puntual. En este articulo utilizamos métodos bayesianos para estimar el modelo factorial dinámico para riesgo de quiebra utilizando datos de calificaciones de riesgo sobre un portafolio crediticio (McNeil et al., 2005; McNeil and Wendin, 2007). Los métodos bayesianos nos permiten:

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incorporar formalmente la información experta en el proceso de estimación de las correlaciones mediante la distribución a priori y obtener intervalos de confianza alrededor de los parámetros de interés. Los resultados indican: i) un modelo de dos factores se ajusta mejor a la información histórica de quiebras, que el modelo de un factor (recomendado en Basilea II), ii) resalta la importancia de la introducción de factores no-observables en la especificación del modelo, en particular, las propiedades estadísticas de los factores no-observables puede tener un efecto importante sobre la magnitud de las correlaciones estimadas, iii) las distribuciones a posteriori de las correlaciones entre los activos indican que los intervalos sugeridos por el documento de Basilea subestiman el riesgo sistémico.

Clasificación JEL: G32, G33, C01.
Palabras clave: Correlación de activos, modelos espacio estado no gausianos, téncicas de estimación bayesiana, modelo binomial con inflación cero.

1 Introduction

Asset dependence in portfolio credit risk management is a topic of growing importance for practitioners and academics. Changes in the most common form of dependence (correlation) across assets transfer some of the risk from the mean towards the tail of the loss distribution. Any increase in correlation between the assets fattens the tail of the loss distribution and therefore requires a greater amount of capital set aside to cover unexpected losses. Hence asset correlation is a cornerstone parameter in the estimation of a bank's capital requirements. Tarashev et al. (2007) show that misspecified or incorrectly calibrated correlations can lead to significant inaccuracies in the measures of portfolio credit risk and economic capital.

The Basel accord of 1988 was a first attempt to establish an international standard on a bank's capital requirements. However a significant drawback was the accord's crude approach to determine the risk weights assigned to different positions in a bank's portfolio. For example, a private firm with a top rating would receive a weight a hundred times higher than any type of sovereign debt, regardless of the rating of the former. The second Basel (henceforth Basel II) corrected the imbalance by accounting for the relative credit quality of the issuers.[1] Under Basel II regulators gave more leeway to banks with the hope that they are able to perform a more accurate measure of the risk heterogeneity of their portfolio. Hence bank managers have greater freedom to calibrate the assigned risk weights and derive more accurate loss distributions for their portfolios.

In general, techniques to derive the loss distribution for the portfolio require simulation. Considering the dependence across all individual names would be very cumbersome, hence factor models provide simple ways to map the dependence structure in the portfolio. Under the internal-rating-based (IRB) approach to determine the risk weights, proposed in Basel II, the underlying structure behind default dependence is a one factor model. Basel II suggests a value for the correlation parameter, the unique dependence parameter of this one factor model, between 0.12 and 0.24. The literature proposes various estimates for these values in the ranges (0.01-0.1) Chernih et al. (2006) and (0.05-0.21) Akhavein et al. (2005).

The risk factor model, used extensively in the literature on dynamic modeling of default risk, provides the structure to estimate asset correlations with rating data. This model decomposes credit risk into systematic (macro related) and idiosyncratic (issuer specific) components. In this context (McNeil et al., 2005; McNeil and Wendin, 2007) and Koopman and Lucas (2008) provide estimation methods to fit the dynamics of default. The former explores heterogeneity among industry sectors and also across rating classes whereas the later only explores heterogeneity across ratings. Both articles recognize two difficulties inherent to rating data and, particularly, default: i) rating transitions are scarce events and ii) defaults are extremely rare events. These two

[1]The recommendations from the Basel II accord are contained in a document published by the Basel Committee on banking supervision (2006).

elements make statistical inference more difficult. However, rating transition data is a preferred proxy for changes in the creditworthiness of issuers because it is more direct than using other proxies such as equity or spread data. Moreover, equity based correlation is not readily available for some types of issuers. For example, for sovereigns and structured products there is no information available for equity or debt. Therefore, it is not possible to link equity and assets through the option-theoretic framework due to Merton (1974).

The aim of this article is twofold: First, estimate asset correlations within and across different identifiable forms of grouping the issuers. Second, provide a sensibility analysis of these estimates with respect to the model assumptions. We use Moody's rating data for Corporate defaults and Structured products. The Corporate default database contains information on 51,542 rating actions affecting 12,292 corporate and financial institutions during the period 1970 to 2009. The Structured products database contains information on 377,005 rating actions affecting 134,554 structured products during the period 1981 to 2009. The database contains information on group affiliation of the issuers such as type of product, or economic sector and country where a firm carries out its business.[2] According to the group affiliation, firms are organized into 11 sectors, 7 world regions and 6 structured products (Table 1).

The disaggregated approach (first objective), with respect to a world aggregate, contributes to the existing literature on asset correlation since most of the literature has focused on estimating these models on aggregate data (in particular aggregate US default count). With respect to world region affiliation and structured products, the results in this article are a novelty. Furthermore, if accounting for heterogeneity in a bank's portfolio is an important part of Basel II, it is senseless estimating models based on the aggregated data. By moving away from the aggregated data, the few historical observations that are available on rating transitions (especially default) become even more sparse. Therefore the existing methodologies encounter problems due to the sparsity of the data.

The sensibility of the estimates of asset correlation with respect to the model assumptions (second objectives) goes beyond the Basel II benchmark: the one factor model. The elements of the model that are analyzed, with regard to their effect over the parameters of interests, are the following: i) introduction of additional group specific factors (*i.e.* a two factor model), ii) the nature for the factors (*i.e.* observed or unobserved), iii) the data generating process of the factors, iv) the functional form of the default probability (*i.e.* probit or logit), and v) for a given rating system, the implications of migrating from different ratings to default (*i.e.* correlation asymmetry).

We use a generalized linear mixed model (GLMM hereafter) for estimation. This model considers the observed number of firms that perform some migration (possibly to the default state), out of a total number of firms within

[2] The main unit of analysis throughout the paper is an issued financial obligation that has some particular rating. The financial obligation can take many forms: On one hand it can be a corporate bond, issued by a firm. On the other hand it can also be a financial product such as a structured product. Therefore, it is important to note that when we refer to firms or issuers, we implicitly refer to the entity which is liable for such financial obligations.

a given group (say an economic sector or world region), as a realization of a binomial distribution conditional on the state of some unobserved systematic factor. A one or two factor model (1-F, 2-F, henceforth) allows the decomposition of default risk into the estimated factor(s) and the idiosyncratic component. A set of identifying assumptions on the model allows for the estimation of both the factor(s) and the factor(s) loading(s). The state space model built from this setup has a measurement equation that has the form of a binomial distribution (making the model non-Gaussian and non-linear).

We find that the loading parameters of the factors across the 11 sectors, 7 world regions and 6 structured products are in general statistically significant. We recover the asset correlations from the factor loadings and observe that in some cases, the asset correlations are higher than the Basel II recommended values. For most models there is even a null or very small probability that the asset correlation parameter is within the bounds recommended in the Basel II document. Asset correlation is in particular very sensitive to the assumptions of the statistical model; for instance if the unobserved component is autoregressive, as opposed to *i.i.d.*. Moreover, the two factor model with AR(1) dynamics for the global factor and with a local systemic factor (called it sectorial, regional, or product) is able to reproduce better the observed number of defaults than the 1-F framework recommended by Basel II.

The results have two direct implications on the measurement of economic capital. First, they show that the one factor model is too restrictive to account in a proper manner for the dependence structure in the data and hence the portfolio. A two factor model provides a hierarchical structure to the banks portfolio while still being parsimonious in terms of the parameters. This model includes a global systemic factor plus a local systemic factor in addition to the idiosyncratic component. The set of local systemic factors account for significant difference across identifiable grouping characteristic within the portfolio such as economic sectors and world regions. Second the estimates of the dependent structure are strongly reliant on modeling assumptions, hence they convey significant model risk. This source of model risk should be taken into account in the process of model validation by the regulators.

The outline of the paper is as follows: Section 2 presents the dynamic default risk model. Section 3 describes the data. Section 4 presents the estimation methods, the results and the implications on the estimation of economic capital. Sections 5 provides methodological solution to the common data scarcity problem found in default models. Section 6 concludes.

2 Dynamic factor model of default risk

The default risk model has its roots in the work by Merton (1974). In the last 10 years this model has been at the center of the literature on portfolio credit risk modeling. The most general version of the Merton model considers the asset value of a firm $i = 1, .., N$ at time $t = 1, .., T$, $V_{i,t}$ as a latent stochastic variable. Let $V_{i,t}$ follow a standard normal distribution. If $V_{i,t}$ falls below a predetermined threshold $\mu_{i,t}$ (related to the level of debt) then a particular event is triggered. This event refers to a transition between states defined under some rating system. For capital adequacy purposes, the most important event is default. However, since historically this is a rare event, it is also interesting to consider a larger state-space to account for all possible transition in a given rating system.[3] These firms belong to the portfolio of an investor (say a bank) that wishes to model the default dependence across the portfolio. With this in mind, the investor considers a F-factor model (F-F, henceforth) as the underlying structure behind the dynamics and dependence structure of the asset value for the firms that belong in the portfolio:

$$V_{i,t} := \sum_{f=1}^{F} a_{f,i} B_{f,t} + \sqrt{1 - \sum_{f=1}^{F} a_{f,i}^2} e_{i,t}, \forall t \in T. \quad (1)$$

In equation (1) the asset value of the firm is driven by F common factors $B_{f,t}$ (common to all firms) and a firm-idiosyncratic component, $e_{i,t}$ (Demey et al., 2004). Let $\mathbf{B_t} = (B_{1,t}, ..., B_{F,t})$ and $\mathbf{e_t} = (e_{1,t}, ..., e_{N,t})$ be two $F \times 1$ and $N \times 1$ vector of factors and idiosyncratic components. We assume that both components follow a multivariate normal distribution, $\mathbf{B_t} \sim N(\mathbf{0}, I_F)$, $\mathbf{e_t} \sim N(\mathbf{0}, I_N)$, and are orthogonal to each other, $E[e_{i,t}, B_{j,s}] = 0 \; \forall t, s, i \neq j$. The elements $a_{f,i}$ make up the factor loading matrix A (of dimension $N \times F$). The weighting scheme of the F-F model along with the distributional assumptions on the factors and idiosyncratic components guarantees that the asset values are standard normally distributed. Furthermore, under these conditions, the entire dependent structure is determined by $AA' = \Sigma$ (a $N \times N$ matrix).

Although the model is indexed for a particular firm, in practice estimation of the parameters is performed on a more aggregate scale. If the parameters are indexed at the firm level then there are a total of $N(N+1)/2$ parameters to estimate the dependence structure. This model is therefore computationally expensive for a large set of firms. One way to reduce dimensionality is to define a set of homogeneous risk classes, i.e. group firms by some identifiable characteristic, economic sector or the world region they belong to. Note $g = 1, \ldots, G$ as group indicator where $G << N$ (much smaller that N), with the following implications for the model parameters:

1. Default threshold is unique within each group and across time, $\mu_{i,t} = \mu_g \; \forall i \in g$.
2. Constant correlation between firms in same group, $\rho_{i,j} = \rho_g \; \forall i, j \in g$.
3. Unique correlation between firms in different groups, $\rho_{i,j} = \rho_{g,d} \; \forall i \in g, j \in d$.

These assumptions imply a symmetric $G \times G$ correlation matrix Σ

$$\Sigma = \begin{pmatrix} \rho_1 & \rho_{1,2} & \cdots & \rho_{1,G} \\ \rho_{1,2} & \rho_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{G-1,G} \\ \rho_{1,G} & \cdots & \rho_{G-1,G} & \rho_G \end{pmatrix}.$$

[3] An example of a rating system is Moody's ratings on long term obligations (broad version): Aaa, Aa, A, Baa (investment grade), Ba, B, Caa, Ca, C(speculative grade or non-investment grade).

With the previous assumptions and if Σ is positive and definite, the F-factor model with $G(G+1)/2$ parameters is:

$$V_{i,t} := \sum_{f=1}^{F} a_{f,g} B_{f,t} + \sqrt{1-\rho_g} e_{i,t}, \forall i \in g. \tag{2}$$

We introduce an additional restriction so as to further reduce the parameter space to $G+1$ correlations. This restriction implies that the correlation among two groups is unique among all the groups, $\rho_{g,d} = \rho \; \forall g \neq d$, which implies

$$\Sigma = \begin{pmatrix} \rho_1 & \rho & \cdots & \rho \\ \rho & \rho_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho & \cdots & \rho & \rho_G \end{pmatrix}.$$

where ρ denoted inter correlation and $\rho_i \; j = 1, ..., G$ intra correlations. We can be show that this new dependence structure is equivalent to the dependence structure derived from a 2-factor model (2-F, henceforth), instead of the F-F model:

$$V_{i,t} := \sqrt{\rho} B_t + \sqrt{\rho_g - \rho} B_{g,t} + \sqrt{1-\rho_g} e_{i,t}, \forall i \in g, \tag{3}$$

where B_t represents a global systemic factor and $B_{t,g}$ represent a local systemic factors that determines the default process.

One further restriction is possible so that there is only one relevant risk class: $\rho_g = \rho$, hence the model reduces to a 1-factor model (1-F, henceforth)

$$V_{i,t} := \sqrt{\rho} B_t + \sqrt{1-\rho} e_{i,t} \forall i. \tag{4}$$

In this case, the dependence structure across a set of firms (in the bank's portfolio) is determined entirely by the parameter ρ. This model reflects the Basel II framework and considered to be an oversimplified representation of the factor structure underlying default dependence, particularly for internationally active banks (McNeil et al., 2005). The main pitfall of the 1-F approach is that it does not detect concentrations or recognize diversification (the dependence structure of the whole portfolio is described by one parameter).

Once a particular dynamic structure for the asset value $V_{i,t}$ is chosen as satisfactory (F-F, 2-F, or 1-F), the next step is to link this setup to the observed rating transitions in order to estimate the parameters of interest *i.e.*, the elements of the Σ matrix. The rating information on the firms, such as the one provided by Moodys, provides the count data to characterize default risk in terms of the number of firms that went into default for a particular period. Let $k_{g,t}$ be the number of firms in group g at time t, and $y_{g,t}$ the number of firms that made some transition between the two states (non-default and default) between t and $t+1$. We assume that the number of defaults are conditionally independent across time given the realization of the latent factors. Then $y_{g,t}$ has the following conditional distribution

$$y_{g,t} | \mathbf{B}_t \sim Binomial(k_{g,t}, \pi_{g,t}), g = 1, .., G; t = 1, .., T, \tag{5}$$

where $\pi_{g,t}$ is the conditional probability of default and $\mathbf{B}_t = (B_t, B_{1,t}, \ldots, B_{G,t})$ is the vector of global and local systemic factors. Equation (3) constitutes the measurement equation of a state space model.

As mentioned previously, default occurs if the asset value of the firm $V_{i,t}$ falls bellow threshold $\mu_{g,t}$. Therefore, this probability can be expressed as a probability function $P : \mathbb{R} \to (0,1)$, depending on the threshold and the dynamics (F-F, 2-F, 1-F) that describe the evolution of the asset value of the firms that belong to the portfolio:

$$\begin{aligned}\pi_{g,t} &= P(V_{i,t} \leq \mu_g \mid \mathbf{B}_t) \\ &= P\left(e_{i,t} \leq \frac{\mu_g - \sqrt{\rho}B_t - \sqrt{\rho_g - \rho}B_{g,t}}{\sqrt{1-\rho_g}}\right) \\ &= \Phi\left(\frac{\mu_g - \sqrt{\rho}B_t - \sqrt{\rho_g - \rho}B_{g,t}}{\sqrt{1-\rho_g}}\right).\end{aligned}$$

The factors are the main drivers of the credit conditions, often considered as a proxies for the credit cycle, Koopman et al. (2009). We consider multiple dynamics for the unobserved factors (autoregressive, random walk, white noise). For now assume that the factor(s) follow an VAR(1) process:

$$\mathbf{B}_t = \Psi \mathbf{B}_{t-1} + \Theta \eta_t, \tag{6}$$

where $\eta_t \sim N(0, I)$, and $\Psi = diag(\psi, \psi_1, \ldots, \psi_G)$,
$\Theta = diag(\sqrt{1-\psi^2}, \sqrt{1-\psi_1^2}, \ldots, \sqrt{1-\psi_G^2})$. The weighting scheme of the VAR(1) process guarantees that each factor is standardized ($B_{g,t} \sim N(0,1)$), as required. This normalization of the factors is important in order to be able to identify the loading parameters ($\frac{\sqrt{\rho}}{\sqrt{1-\rho_g}}, \frac{\sqrt{\rho_g-\rho}}{\sqrt{1-\rho_g}}$) and the parameters of interest the implied correlations (ρ, ρ_g). Equation 6 constitutes the state equation of a state-space model.

In the portfolio credit risk literature various authors (Gordy and Heitfield, 2002; Demey et al., 2004; Koopman and Lucas, 2008; Wendin and McNeil, 2006; McNeil and Wendin, 2007) propose similar types of factor models for default risk, the so called structural type models that follow the Mertonian framework. All the underlying models, as well a the model proposed in expressions 5 and 6, are special cases of the generalized linear mixed model (GLMM) for portfolio credit risk, see Wendin (2006).

There are two considerations that border on the theoretical and the empirical with respect to the factors, ($\mathbf{B}_t$), that make up the state equation of the model 6. The first issue is whether the factor(s) in the default risk model should be considered as unobserved, observed or both. The factor(s) represent the main driver (systemic component) behind the possibility that a firm goes into default or not. Systematic credit risk factors are usually considered to be correlated with macroeconomic conditions. Nickell et al. (2000), Bangia et al. (2002) Kavvathas (2001), and Pesaran et al. (2006) use macroeconomic variables as factors in default risk models. However, there are some doubts whether there is an adequate alignment between the credit cycle (implied by rating and default data) and the macroeconomic variables. Koopman et al. (2009) indeed show that business cycle, bank lending conditions, and financial market variables have low explanatory power with respect to default and rating dynamic. Das et al. (2007) find that US corporate default rates between 1979 and 2004 vary beyond what can be explained by a model that only includes observable covariates. Such results give a strong motivation for the introduction of unobserved components in default risk models. Furthermore, Wendin and McNeil (2006); McNeil and Wendin (2007) and Koopman and Lucas (2008) show that there are gains in term of the fit of the model and forecasting accuracy, when both observed macroeconomic covariates and unobserved components are considered.

The last issue is the dynamic characterization of the factors. In some of the earliest articles that focus on the estimation of asset correlations, the factors were considered as (*i.i.d.*) standard normal random variables.[4] However, Bangia et al. (2002) and Nickell et al. (2000) have empirically shown that changes in the macroeconomic environment have some effect over rating transitions and default, which suggest that the credit default process is serially correlated. Furthermore, the source of this serial correlation is the autocorrelation present in the factor (observed macro covariates and/or the unobserved component). The existence of serial correlation also points to the fact that rating procedures within the rating agencies are more through-the-cycle than point-in-time.

3 Data and Stylized Facts

3.1 Data

We use Moody's Corporate Default database on issuer senior rating, which contains information on 51,542 rating actions affecting 12,292 corporate and financial institutions during the period 1970 to 2009. [5] We also use Moody's database on Structured Products that contains information on 377,005 rating actions affecting 134,554 structured products (only the super senior trench) during the period 1981 to 2009.

Moody's database considers 9 broad ratings (Aaa, Aa,..) for the period 1970 to 1982 and 18 alphanumeric ratings (Aaa, Aa1, Aa2,..) from 1983 onwards. For consistency the 9 broad ratings are considered throughout the sample. Although Moody's does not have an explicit default state, it does have a flag variable that indicates when an issuer can be considered in default or close to it. Since there are different definitions of default (*e.g.* missed or delayed disbursement of interest and/or principal, bankruptcy, distressed exchange) Moody's keeps rating the issuer according to their rating grades. We use the flag variable to determine a unique date of default irrespective of the fact that Moody's still gives a broad grade.

Additional to the rating information for each issuer there is an assigned country and economic sector codes (two and four digits SIC codes and Moo-

[4] See Gordy and Heitfield (2002) and Demey et al. (2004).

[5] The last observation in the database is April 2009.

Table 1. Description of independent variables

Moodys	Code	y_g	k_g
Sectors			
Banking	BAN	63	17,260
Capital industries	CAI	370	19,477
Consumer industies	COI	291	11,974
Energy & environment	EAE	108	7,429
Finance, insurance & real estate	FIR	47	11,360
Media & publishing	MED	46	1,791
Retail & distribution	RET	137	4,797
Sovereign & public finance	SOV	26	6,364
Technology	TEC	181	9,970
Transportation	TRA	82	4,229
Utilities	UTL	28	11,087
World Regions			
Western Europe	WEP	80	16,717
Eastern Europe	EEP	12	806
North America	NOA	1,104	74,229
Central & South America	SCA	63	4,194
Asia & Oceania	AOC	45	8,695
Middle East	MDE	2	298
Africa	AFK	4	170
Structured Products			
Asset-backed security	ABS	75	53,558
Collateralized debt obligations	CDO	975	31,087
Comm and other mortgage backed security	CMBS	116	35,361
Home Equity Loans	HEL	157	47,993
Other Structured Products	OSP	131	107,831
Residential mortgage backed security	RMB	2,032	135,313

Note: Organization of the Moody's databases according to groups of economic sectors, world regions and structured product types. y_g denotes the total number issuers that have defaulted within each group over the sample period. k_g denotes the total number of issuers within each group across the entire period.

Source: Author's compilation.

dy's own sector codes). We recode the countries into 7 world regions and use Moody's 11 classification as the relevant set of economic sectors (Table 1). We organize the structured products by the type of deal.

The data in the models are the yearly time series of: i) the total number of firms in group g that at time t hold some particular rating, $k_{g,t}$, and ii) the number of firms belonging to that same group g that at the end of time t have defaulted, $y_{g,t}$. These two variables are the only observables in the state space models considered at the end of the previous section.

As mentioned previously, the event of default is an extremely rare event, inference on this type of event is complex. In particular the time series of defaults suffer of overdispersion or zero-inflation. This zero inflation is exacerbated when we disaggregate the data into groups (*i.e.* sectors, regions and products). Since default is already a rare event in the aggregate, then when we make a subgroup of this aggregate, the observed defaults become even fewer within each group. The problem of zero-inflation affects the model since it deviates from the assumption that the default counts have a binomial behavior. In other words, an increasing number of zeros may degenerate the distribution. In order to overcome the problems due to overdispersion, a zero-inflated binomial model for the default counts is developed and estimated in Section 5.

3.2 Stylized Facts

The database on long term corporate issuers is to a great extend (especially in the first part of the sample) composed of US issuers. The US data represents about 65% of the potential data on rating transitions. Figures 1 illustrates the number of defaults for five sectors, consumer industries, and technology, which have a significant number of defaults, banking, finance/insurance/real estate and sovereign/public finance, which have few rating movements. The later illustrates the problem of zero-inflation. Consumer industries have an important participation on defaults throughout all the sample, while technology is a late starter and shows increasing activity in 2002 and 2001, which was a very volatile period for the industry (the burst of the telecommunications bubble that had its peak in the late 2000). Although sectors are believed to show their own dynamics, there are periods of general turmoil (clustering effects) that are evident in the sample, especially at the end of the nineties.

For structured products (Figure 2), the information available has increased with the rapid expansion of the market for these types of securities. Most of the relevant information is at the end of the sample. It is also evident the increasing number of defaults in 2008 and the preliminary data of 2009, especially in collateralized debt obligations and residential mortgage backed securities.

In general the figures on sectors and structures products show a great deal of heterogeneity in default events. In the estimation part, the objective is to try to capture the intra and inter correlations due to rating movements. Furthermore, since rating movements are closely related to creditworthiness, results will give some idea of the asset correlations.

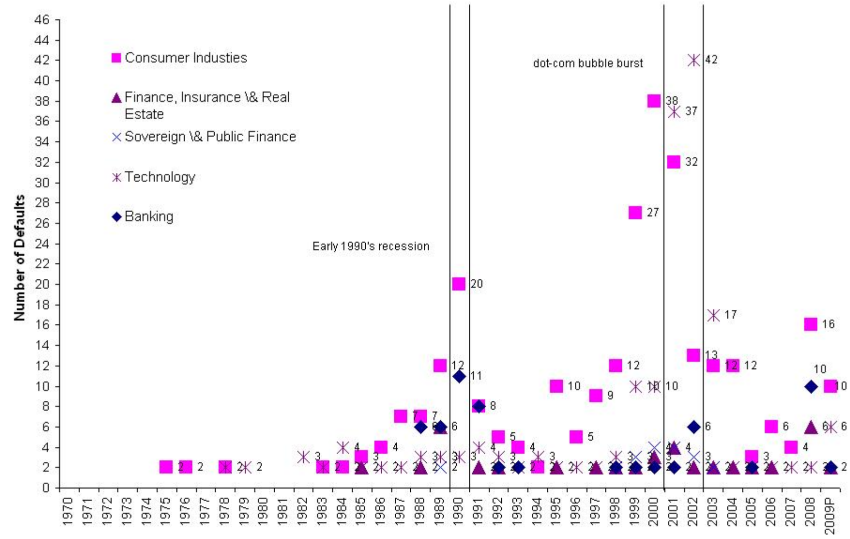


Figure 1. Number of Defaults for some sectors.
Source: Author's compilation.

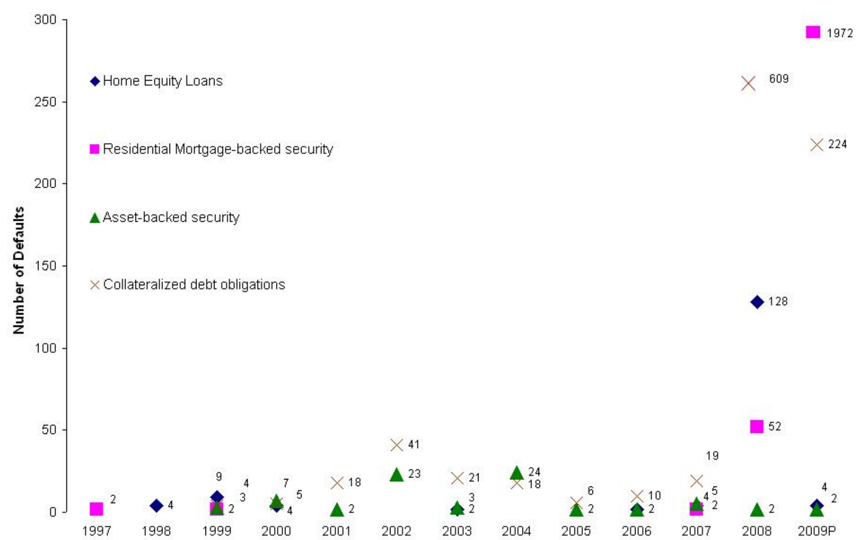


Figure 2. Number of Defaults for some structured products.
Source: Author's compilation.

4 Estimation

The model in section 3 is a non-linear and non-Gaussian state space model because of the binomial form of the measurement equation. Furthermore the model incorporates an unobserved component. In this context, standard linear estimation techniques are not appropriate. The estimation of such model, has been performed on credit rating data, either using a Monte Carlo maximum likelihood method (Koopman and Lucas, 2008; Durbin and Koopman, 1997) or using Bayesian estimation, in particular Gibbs sampling (Wendin, 2006; McNeil et al., 2005; McNeil and Wendin, 2007). This article follows the second approach mainly on two accounts: it provides greater flexibility in dealing with the over dispersion (zero-inflation) and, it is possible to derive a distribution for the asset correlations (parameter of interest).

Denote $\psi := (\psi_1, \ldots, \psi_G)$ as the relevant set of parameters. This set naturally includes the unobserved components. Bayesian inference considers the unknown parameters ψ as random variables with some prior distribution $P(\psi)$. The prior distribution along with the conditional likelihood of the observed data $x := \{(y_{g,t}, k_{g,t})\}_{g=1,t=1}^{G,T}$ are used to derived the posterior distribution for the unknown parameter: $P(\psi \mid x) \propto P(x \mid \psi)P(\psi)$. In some cases the posterior distribution is unattainable analytically. Hence the evaluation of the joint posterior requires the use of Markov chain Monte Carlo (MCMC) algorithms, such as the Gibbs sampler. The Gibbs sampler is a specific componentwise Metropolis-Hastings algorithm that performs sequential updating of the full conditional distributions of the parameters in order to reproduce the joint posterior of the parameters. In other words, the algorithm proceeds by updating each parameter ψ_j by sampling from its respective conditional distribution, given the current values for all other parameters $\psi_{-\mathbf{j}} := (\psi_1, .., \psi_j - 1, \psi_j + 1, .., \psi_J)$ and the data. This conditional distribution is the so-called full conditional distribution (see appendix). With a sufficiently large number of repetitions, it can be shown that, under mild conditions, the updated values represent a sample from the joint posterior distribution.

Each model is estimated using three parallel Markov chains that are initiated with different starting values. Convergence of the Gibbs sampler is assessed using the Gelman and Rubin (1992, 1996) scale-reduction factors. The autocorrelations of sample values are also checked to verify that the chains mix well. Only after convergence, the Deviance Information Criterion (DIC) is used to choose among the different models fitted to the data, following Spiegelhalter et al. (2002). The different model characterize different assumptions for the dynamic factor model for default risk (e.g. 1-F vs 2-F, dynamics of unobserved factors). This criterion resembles the Akaike's Information Criteria, since it is expected to choose the model which has the best out-of-sample predictive power.[6] The DIC is defined as follows. First recall the usual definition of the deviance, $dev = -2logP(x \mid \psi)$. Let $\overline{dev}$ denote the posterior mean of the deviance and $\widehat{dev}$ the point estimate of the deviance computed by sub-

[6] The estimation was performed using WINBUGS Release version 1.4.3, http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/contents.shtml.

stituting the posterior mean of $\widehat{\psi}$. Thus $\widehat{dev} = -2logP(x \mid \widehat{\psi})$. Denote by pD the effective number of parameters (elusive quantity in Bayesian inference) defined as the difference between the posterior mean of the deviance and the deviance of the posterior means, $pD = \overline{dev} - \widehat{dev}$. The DIC is defined as follows: $DIC = \overline{dev} + pD$. The model with the smallest DIC value is considered to be the model that would predict a dataset of the same structure as the data actually observed. Since the distribution of the DIC is unknown (no formal hypothesis testing can be done) it is a difficult task to define what constitutes an important difference in DIC values. Spiegelhalter et al. (2002) propose the following rule of thumb: if the difference in DIC is greater than 10, then the model with the larger DIC value has considerable less support than the model with the smallest DIC value.

As mentioned previously, a further advantage of using a full Bayesian approximation is that the parameters of interest, the asset correlations, and especially the uncertainty about them can be directly obtained from the MCMC. The parameters of the statistical model are the factor loadings, but we know that through a series of identifying restrictions in the economic model, we can establish a functional relationship between the factor loadings and the asset correlations. We can include such function in the Monte Carlo procedure so as to derive directly the parameters of interest and, in particular, derive a confidence set for the implied correlations.

An informative prior is used for the autoregressive coefficient that determines the dynamics of the unobserved factor, $\psi \sim U(-1, 1)$. According to this prior the unobserved factor follows a stationary AR(1) process. Diffuse but proper priors are considered for all other parameters (the factor loadings $\frac{\sqrt{\rho_g - \rho}}{\sqrt{1-\rho_g}} \sim U(0, 10)$, and the default threshold $\mu_g \sim N(0, 10^3)$), however other priors are also possible if specific prior information is available for some parameters.[7][8] The sampling method used for the distributions is a Slice sampler. The sampler was run for 10,000 iterations, with the first 5,000 iterations discarded as a burn-in period.

The aggregate data only has a one factor representation (4), which we denote as Model A in the tables. In such one factor representation the unobserved factor B_t has two possible dynamics a stationary and univariate AR(1) process or a white noise,*i.i.d.* $N(0, 1)$.

Models which we denote B, C and D, in the tables, are based in the panel structure for sectors, regions or structured products. These models have both a one or two factor representation (4, 3). In the two factor representation, the second factor can be thought as a local systemic factor.

The main difference between models C and D is that in the former the first factor $B_t \sim N(0, 1)$ is a stationary AR(1) process. This factor represents the so-called global systemic factor. The second factor of each of the groups is a

[7]This range for the factor loading captures all the possible values for the asset correlation $\rho \in (0, 1)$. Larger interval values, such as an improper prior like $U(-200, 200)$ or $N(0, 10^3)$ only improve decimal point accuracy. The value is also restricted to be positive in order to prevent label switching.

[8]See Tarashev et al. (2007) for some possible informative priors for the some of the parameters.

white noise $B_{t,g} \sim iidN(0,1)\ \forall g$. Note that this second factor represents the local systemic factor and is drawn from a distribution that is unique across all of the groups; whereas in Model D all of the factors are considered to be white noise, therefore the state equation is uniquely determined by the following expression $\mathbf{B}_t = \eta_t$. It is important to note that in every case the unobserved factors are orthonormal, guaranteeing that the factor(s) loading(s) are identified.

4.1 Results

With the aggregate default data from the US, we estimate model A. The value of the persistence parameter ϕ of the AR(1) specification is 0.91 and indicates a strong persistence, as found by Koopman and Lucas (2008), whereas Wendin and McNeil (2006) find smaller values 0.68.[9] The value of the loading parameter is 0.56 and it is close to the estimate obtained for the same specification by Koopman and Lucas (2008). The value of the parameter gives an estimate for asset correlation of 0.24. In general, the estimated parameters are very close to those obtained from a similar set up by Koopman and Lucas (2008). If the unobserved component is assumed to be *i.i.d.* the asset correlation falls to 0.10. The information criteria, DIC, indicates no significant difference with respect to the response function. Furthermore, the model with an AR(1) type unobserved factor performs better than the *i.i.d.* according to the information criteria.[10] Both the AR(1) specification as well as the *i.i.d.* for the unobserved component provide an adequate fit to the aggregate US default data.

Correlation asymmetry is a phenomenon found consistently in the literature irrespective of the data or methodology used in obtaining estimates of asset dependence with defaults (Das and Gengb, 2004). This phenomenon has a further intuitive appeal since it indicates that high graded issuers (in many cases large firms) have a larger exposure to systemic risk, whereas low graded issuers (medium and small firms) face more idiosyncratic risk. Using the aggregate data for all world regions (not only US) the estimated asset correlations, from model A for the AR(1) and *i.i.d.* specifications for the unobserved component are 0.18 and 0.09, respectably (Figure 3).

Three separate exercises were also considered using model A. The first only considers defaults from non-investment grade issuers. The second only considers defaults from investment grade issuers (Table 2). Results are consistent with correlation asymmetry, as they indicate an inverse relation between correlation and the quality of the issuer. In other words correlation is larger for investment grade issuers ($\rho_{IG} = 0.4$) than for non-investment grade issuers ($\rho_{NIG} = 0.17$) issuers (Figure 3).

[9] The majority of the Moody's data comes from the US specially the data before 1990.

[10] Higher order autoregressive process of the unobserved factor were tested at some point but provide no improvement over the AR(1). Results are available under request.

Table 2. Estimation Results from the Aggregate Corporate default data from 1970 to 2009.

	Non-Default to Default		Non-Investment Grade to Default		Investment Grade to Default	
	Model A AR(1)	Model A *i.i.d.*	Model A AR(1)	Model A *i.i.d.*	Model A AR(1)	Model A *i.i.d.*
Loadings	0.4593^b	0.3123^a	0.4558^b	0.3054^a	0.8291^a	0.748^a
	(0.183)	(0.312)	(0.202)	(0.0446)	(0.126)	(0.148)
ψ	0.8743^a		0.8459^a		0.7085^a	
	(0.874)		(0.106)		(0.126)	
DIC	160	257	164	254	96	100
Asset Correlations						
2.5	0.053	0.057	0.050	0.051	0.223	0.166
Mean	0.178	0.090	0.177	0.086	0.404	0.356
97.5	0.450	0.138	0.472	0.145	0.497	0.492

Note: The results are first taken with respect to all rated firms, second only taking non-investment grade rated firms and last only taking investment grade rated firms. Results are presented only for the Probit response function. The Monte Carlo standard errors of the mean are shown in parentheses. ^{c}, ^{b}, and ^{a} denote significance at the 10%, 5%, and 1% levels, respectively.

Source: Author's estimation.

The last exercise based on the model A, provides a simplified approach to capture the effects of tail risk. Since model A is a 1-F model tail risk is naively introduced by changing the standard assumption on the distribution of the unobserved factor.[11] The unobserved factor, for the sole purpose of this exercise, follows a standardized Student-t distribution with two degrees of freedom, $B_t \sim iid\ t(0,1,2)$. The results indicate no significant gain with respect to the standard assumptions on the dynamics of the unobserved factor, according to DIC. However, the estimated value for the correlation is approximately half the value obtained when using the standard assumptions on the dynamics of the unobserved factor. These results imply that by considering a distribution with thicker tails than the standard Gaussian, this model is able to capture the same default dynamics as the standard model, requiring on average a smaller estimate of the asset correlation.

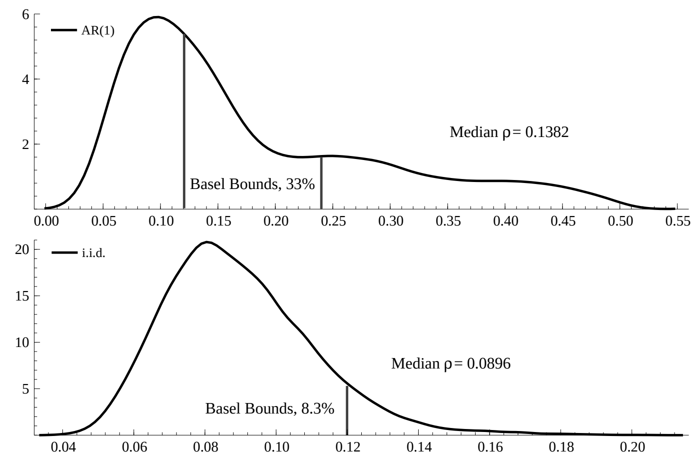
Different types of factor models (models A through D) are considered for the panel data of sectors (3), regions (table 4) and structured products (table 5). The tables report the posterior means and the standard errors for the model parameters using the Probit type response function.[12] For the parameters in each model, on average convergence of the Markov chains was reached after 4,000 to 6,000 iterations.

In general, the parameters indicate that the global systemic factor loading is statistically significant in all the models (Tables 3 to 5), except for model D in the sectors and regions. On the other hand, the local systemic factor weight is not always statistically significant. For example, the extreme case is model D for regions, where the overall factors (except in the case of Western Europe) do not play a role in the dynamics of defaults.

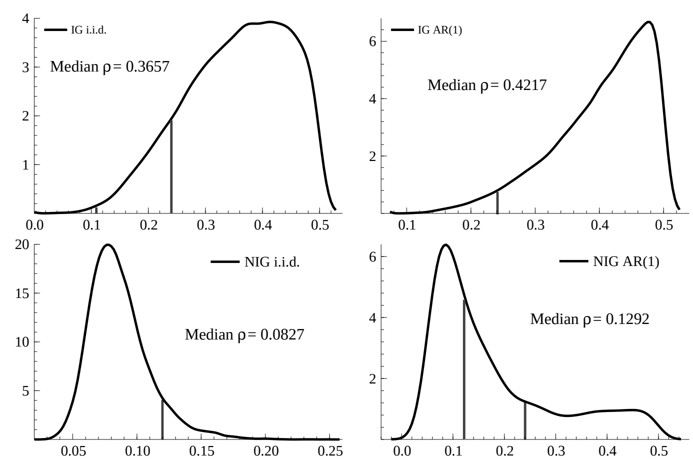
Once convergence is obtained, the DIC indicates that the most appropriate model is model C for sectors and regions, and model D for products. Model C is a two factor model that has an autoregressive global unobserved factor and a unique *i.i.d.* component for the local systemic factor. This previous specification is the same as model 4 of McNeil and Wendin (2007) and it is one of the models used to derived the asset correlations (Table 6). Model D has the same structure as the model estimated in Demey et al. (2004) and the asset correlations derived from it are presented in Table 7. There is a striking difference between the asset correlations derived from both models. Asset correlations derived from model C are much higher than the ones derived from model D, particularly in the case for sectors and regions. While the asset correlations of model D are within the range of the Basel II (2006) recommended values (0.12,0.24), even lower in some cases, model C proposes much higher values, with a possible range of 0.3 to 0.5. Another way to look at it is to determine the probability that the estimated asset correlation is within the Basel II bounds,

[11] Accounting for tail risk would also require to change the distribution of the firm-idiosyncratic component, $e_{i,t}$. This has an effect on the choice of link function. In order to avoid this additional complication only the distribution of the unobserved factor is modified. It is important to note that the change is enough to loose the tractability of the distribution of the asset value, $V_{i,t}$.

[12] There were no significant differences between the response functions, only the Probit results are presented, but the Logit results are available upon request.



(a) All ratings



(b) Investment & Non-Investment Grade

Figure 3. Estimated Bounds vs. the Basel II recommended bounds. Posterior distribution of implied asset correlations from a one-factor model.
Note: Implied asset correlation depends critically on model assumptions.
Source: Author's estimation.

$P(0.12 \leq \rho \leq 0.24)$, using the posterior distribution. Whereas in model C the probabilities is well below 7% for regions, and for sectors and products they are zero, in model D these same probabilities are 11% for regions and sectors and 5% for products. These results indicate that the Basel recommended bounds are overoptimistic with respect to uncertainty surrounding asset dependence. Furthermore, in most of the models considered in this Section, the Basel recommended bounds are located in the left tail of the posterior distri-

Table 3. Estimation Results from the desegregated panel of economic sector default data from 1970 to 2009.

Model		A	B	C	D
		0.878^a		0.845^a	0.170^c
		(0.117)		(0.147)	(0.094)
Specific Loadings					
	BAN		0.809^a	0.467^a	0.446^a
			(0.128)	(0.118)	(0.110)
	CAI		0.773^a	0.071^c	0.074
			(0.0989)	(0.041)	(0.045)
	COI		0.723^a	0.101^b	0.178^a
			(0.104)	(0.051)	(0.060)
	EAE		0.765^a	0.412^a	0.128^c
			(0.126)	(0.096)	(0.077)
	FIR		0.577^a	0.177^c	0.087
			(0.163)	(0.100)	(0.064)
	MED		0.463^b	0.292^c	0.120^c
			(0.181)	(0.158)	(0.070)
	RET		0.636^a	0.106	0.082
			(0.123)	(0.069)	(0.058)
	SOV		0.774^a	0.108	0.203^a
			(0.152)	(0.089)	(0.060)
	TEC		0.908^a	0.259^a	0.225^a
			(0.075)	(0.056)	(0.068)
	TRA		0.517^a	0.272^b	0.095
			(0.139)	(0.108)	(0.067)
	UTL		0.706^a	0.236^c	0.192^b
			(0.162)	(0.136)	(0.093)
ψ		0.962^a	0.950^a	0.956^a	
		(0.0223)	(0.020)	(0.036)	
DIC		1520.5	1522.1	1341.8	1360.9

Note: Results are presented only for the Probit response function. The Monte Carlo standard errors of the mean are shown in parentheses. ^{c}, ^{b}, and ^{a} denote significance at the 10%, 5%, and 1% levels, respectively.

Source: Author's estimation.

bution of asset correlation hence they seem to be consistent with a low level of systemic risk (figure 4).

The differences in the implied asset correlations due to the dynamics of the unobserved factors do not seem to be only an empirical issue, but theoretical as well. It also makes intuitive sense because even though the factors are stationary, the fact that there is persistence (in some cases it is very strong), implies that any shocks in the short run will not dissipate from year to year. This also can explain the clustering phenomenon (across sectors and regions) of the number of defaults that is observed in the stylized facts. The asset corre-

Table 4. Estimation Results from the desegregated panel of world region default data from 1970 to 2009.

Model		A	B	C	D
Global		0.781^a		0.792^a	0.178^c
		(0.163)		(0.182)	(0.107)
Specific Loadings					
	WEP		0.553^a	0.271^b	0.244^b
			(0.139)	(0.113)	(0.115)
	EEP		0.746^a	0.281	0.113
			(0.180)	(0.205)	(0.084)
	NOA		0.588^a	0.052	0.081
			(0.115)	(0.039)	(0.053)
	SCA		0.685^a	0.238^b	0.109^c
			(0.155)	(0.102)	(0.065)
	AOC		0.787^a	0.308^b	0.154^c
			(0.141)	(0.150)	(0.093)
	MDE		0.380	0.589^b	0.114
			(0.259)	(0.256)	(0.081)
	AFK		0.806^a	0.615^b	0.173^c
			(0.167)	(0.241)	(0.104)
ψ		0.952^a	0.929^a	0.948^a	
		(0.034)	(0.035)	(0.046)	
DIC		543.1	562.1	494.8	509.4

Note: Results are presented only for the Probit response function. The Monte Carlo standard errors of the mean are shown in parentheses. ^{c}, ^{b}, and ^{a} denote significance at the 10%, 5%, and 1% levels, respectively.

Source: Author's estimation.

lations (Tables 6 and 7) also indicate (consistently across methods) that there are some sectors that are more volatile than others such as banking, energy and technology.

4.2 Implications on economic capital

As briefly mentioned in the introduction very strong modeling assumptions in credit portfolio models carry an important impact on economic capital. Tarashev et al. (2007) present an empirical procedure for analyzing the impact of model misspecification and calibration errors on measures of portfolio credit risk. A large part of the analysis presented in Tarashev et al. (2007) is focused on the Basel II benchmark for the IRB approach to determine determine the risk weights: the Asymptotic single-risk factor model (ASRF). It is well known that this model has two very strong assumptions that are often criticized as sources of misspecification errors. First, the model assumes that the systemic component of credit risk is governed by a single common factor. Second, the model assumes that the portfolio is perfectly granular such that all idiosyn-

Table 5. Estimation Results from the desegregated panel of structured products default data from 1982 to 2009.

Model		A	B	C	D
Global		0.949^a		0.943^a	0.754^a
		(0.044)		(0.050)	(0.229)
Specific Loadings					
	ABS		0.008	0.446^a	0.617^a
			(0.007)	(0.124)	(0.140)
	CDO		0.193^a	0.335^a	0.446^a
			(0.011)	(0.099)	(0.136)
	CMBS		0.155^a	0.317^c	0.647^a
			(0.020)	(0.165)	(0.149)
	HEL		0.146^a	0.645^a	0.364
			(0.016)	(0.170)	(0.241)
	OSP		0.066^a	0.354^b	0.827^a
			(0.012)	(0.156)	(0.118)
	RMB		0.985^a	0.818^a	0.741^a
			(0.015)	(0.127)	(0.213)
ψ		0.918^a	0.641^a	0.932^a	
		(0.021)	(0.061)	(0.018)	
DIC		593.0	464.6	301.1	298.7

Note: Results are presented only for the Probit response function. The Monte Carlo standard errors of the mean are shown in parentheses. ^{c}, ^{b}, and ^{a} denote significance at the 10%, 5%, and 1% levels, respectively.

Source: Author's estimation.

cratic risk are diversified away. Furthermore, even if the model is well specified there are additional sources of uncertainty that stem form the calibration of the correlation among the assets. Large uncertainty over the estimated factor weights can induced large variations over the implied economic capital. Their results indicate that errors in calibration, as opposed to errors in specification, of the ASFR model are the main sources of potential uncertainty of credit risk in large portfolios. Where as the single factor specification and the granularity effect under predict the target capital by less than 5%, calibration errors may induce over(under)predict the target level by 8% for each percentage point over(under)estimation of the average correlation coefficient.

Using a similar procedure as in Tarashev et al. (2007) this section evaluates the effects over economic capital that arise from two sources: First, the effect of considering a 1-F model rather than a 2-F model. Second, the uncertainty with respect to the estimated asset correlations. Since we are not interested in the granularity effect, we stay in the asymptotic factor model framework. Recall expression 3, the specification of the 2-F model, such expression determines the dynamics of the asset value of a firm that belongs in group g. The 2-F

Table 6. Asset correlations obtained from the default risk model. Model type C and Probit response function.

Model C	2.5	Mean	97.5
Global	0.150	0.329	0.396
Banking	0.283	0.482	0.583
Capital industries	0.182	0.415	0.500
Consumer Industies	0.187	0.417	0.501
Energy & Environment	0.267	0.469	0.560
Finance, Insurance & Real Estate	0.198	0.427	0.516
Media & Publishing	0.230	0.447	0.555
Retail & Distribution	0.189	0.418	0.503
Sovereign & Public Finance	0.188	0.419	0.506
Technology	0.215	0.436	0.520
Transportation	0.221	0.441	0.534
Utilities	0.206	0.437	0.538
Global	0.088	0.301	0.392
Western Europe	0.154	0.412	0.534
Eastern Europe	0.139	0.420	0.582
North America	0.105	0.383	0.498
Central & South America	0.148	0.406	0.521
Asia & Oceania	0.153	0.421	0.553
Middle East	0.227	0.499	0.642
Africa	0.241	0.506	0.643
Global	0.310	0.367	0.391
ABS	0.442	0.523	0.593
CDO	0.442	0.502	0.552
CMBS	0.430	0.502	0.577
HEL	0.476	0.567	0.651
OSP	0.415	0.507	0.591
RMB	0.514	0.608	0.660

Source: Author's estimation.

model can be expressed such that it nest the 1-F model:

$$V_{i,t} := \sqrt{\rho}B_t + \sqrt{\Delta\rho_g}B_{g,t} + \sqrt{1-(\rho+\Delta\rho_g)}e_{i,t}, \forall i \in g, \tag{7}$$

where $\Delta\rho_g = \rho_g - \rho$ measures the difference between inter and intra correlation. By definition $\Delta\rho_g \geq 0$. This implies that there is some diversification affect by holding positions in different groups (sectors or world regions) rather than concentrating all exposures in a particular group. The limit of the diver-

Table 7. Asset correlations obtained from the default risk model. Model type D and Probit response function.

Model D	2.5	Mean	97.5
Global	0.000	0.032	0.093
Banking	0.080	0.193	0.348
Capital industries	0.002	0.042	0.111
Consumer Industies	0.013	0.066	0.146
Energy & Environment	0.003	0.055	0.138
Finance, Insurance & Real Estate	0.002	0.046	0.123
Media & Publishing	0.003	0.053	0.134
Retail & Distribution	0.002	0.045	0.119
Sovereign & Public Finance	0.018	0.075	0.153
Technology	0.022	0.083	0.174
Transportation	0.003	0.047	0.124
Utilities	0.032	0.076	0.146
Global	0.000	0.036	0.112
Western Europe	0.010	0.099	0.233
Eastern Europe	0.002	0.057	0.153
North America	0.001	0.048	0.133
Central & South America	0.002	0.054	0.142
Asia & Oceania	0.004	0.068	0.157
Middle East	0.002	0.057	0.150
Africa	0.010	0.076	0.158
Global	0.025	0.277	0.386
ABS	0.229	0.489	0.624
CDO	0.143	0.437	0.572
CMBS	0.276	0.501	0.620
HEL	0.056	0.416	0.615
OSP	0.341	0.556	0.655
RMB	0.299	0.538	0.642

Source: Author's estimation.

sification effect is determined by the global systemic risk (non-diversifiable risk). If $\Delta\rho_g = 0$ then we obtain the 1-F model and this means that there are no further gains obtained by holding positions across different groups of firms.

Following Tarashev et al. (2007), let $\mathbf{1}_{V_{i,t} \leq \mu_g}$ denote an indicator variable that is equal to 1 if firm $i \in g$ is in default at time t and, 0 otherwise. By taking expectations over the indicator variable and assuming that the asset value dynamics is given by 7, we arrive at the conditional probability of default,

$$\begin{aligned} E\left[\mathbf{1}_{V_{i,t}\leq\mu_g}\right] &= P(V_{i,t}\leq\mu_g) \\ &= P\left(e_{i,t}\leq\frac{\mu_g-\sqrt{\rho}B_t-\sqrt{\Delta\rho_g}B_{g,t}}{\sqrt{1-(\rho+\Delta\rho_g)}}\right) \\ &= \Delta\left(\frac{\mu_g-\sqrt{\rho}B_t-\sqrt{\Delta\rho_g}B_{g,t}}{\sqrt{1-(\rho+\Delta\rho_g)}}\right). \end{aligned}$$

Under the ASF model (perfect granularity), the Law of Large Numbers implies that the conditional total loss on the portfolio, $TL|B, B_g$, is deterministic for given values of the state variables, B, B_g:[13]

$$\begin{aligned} TL|B,B_g &= \sum_i w_i E\left[LGD_i\right] E\left[\mathbf{1}_{V_{i,t}\leq\mu_g}\right] \\ &= \sum_i w_i E\left[LGD_i\right]\Phi\left(\frac{\mu_g-\sqrt{\rho}B_t-\sqrt{\Delta\rho_g}B_{g,t}}{\sqrt{1-(\rho+\Delta\rho_g)}}\right), \end{aligned}$$

where w_i is the weight of the exposure to firm i, LGD_i is the Loss Given Default for firm i and $\Phi(.)$ is the cumulative distribution function of a standard normal random variable. The unconditional loss distribution can be obtained from the previous expression by recalling the distributional assumption on the conditioning factors. From section 2 we know that all of the factors are distributed as standard normal random variables. Therefore, the $1-\alpha$ level credit VaR or the $(1-\alpha)^{th}$ percentile of the distribution of total losses is:

$$TL_{1-\alpha} = \sum_i w_i E\left[LGD_i\right]\Phi\left(\frac{\mu_g-\sqrt{\rho}\Phi^{-1}(\alpha)-\sqrt{\Delta\rho_g}\Phi^{-1}(\alpha)}{\sqrt{1-(\rho+\Delta\rho_g)}}\right),$$

where $\Phi^{-1}(\alpha)$ is the α^{th} percentile in the distribution of the factors. To cover unexpected losses with probability $(1-\alpha)$ the capital for the entire portfolio is:

$$\begin{aligned} \kappa &= TL_{1-\alpha} - \sum_i w_i E\left[LGD_i\right] PD_i \\ &= \sum_i w_i E\left[LGD_i\right]\left[\Phi\left(\frac{\mu_g-\sqrt{\rho}\Phi^{-1}(\alpha)-\sqrt{\Delta\rho_g}\Phi^{-1}(\alpha)}{\sqrt{1-(\rho+\Delta\rho_g)}}\right)-PD_i\right], \end{aligned}$$

where PD_i is the expected probability of default for exposure i.

Since the interest of the exercise is on how different values for ρ and $\Delta\rho_g$ capture either specification issues (i.e. 1-F vs. 2-F) or the uncertainty regarding the estimation of asset correlation, some simplifications are in order. We

[13] For convenience from this point on all time subscripts are suppressed and the respective link function is assumed to be of the probit type.

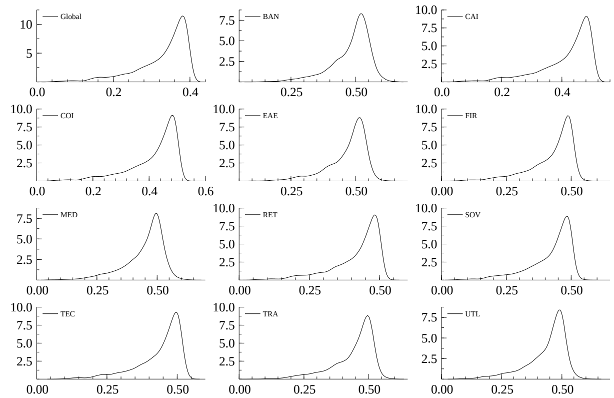
eliminate the weights w_i by assuming equal exposure, LGD's and PD's across firms, $LGD_i = 45\%$ and $PD_i = 1\% \; \forall i$. Therefore the percentage of capital,

$$\kappa = E\left[LGD\right]\left[\Phi\left(\frac{\mu_g - \sqrt{\rho}\Phi^{-1}(\alpha) - \sqrt{\Delta\rho_g}\Phi^{-1}(\alpha)}{\sqrt{1-(\rho+\Delta\rho_g)}}\right) - PD\right],$$

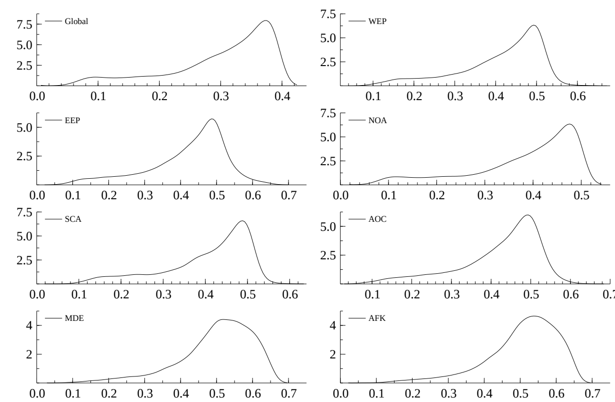
where the threshold $\mu_g = -2.37$ is consistent with PD. $\rho \in [0.15, 0.396]$ and $\Delta\rho_g \in [0, 0.187]$, these values are consistent with the 2.5^{th} and 97.5^{th} quantiles of the correlations for sectors presented in table 8.[14]

The results are presented in figure 5. First, as also indicated by Tarashev et al. (2007), in this particular type of default risk models the economic capital implied by a 2-F model is strictly larger than the economic capital implied by the 1-F model. Since the 2-F model nest the 1-F model (when $\Delta\rho_g = 0$) we observed that for any feasible value of the asset correlations the capital measure is larger in the 2-F specification. Second, the diversification effects from holding different groups of exposures are exhausted as the intra asset correlation belonging to a particular group reaches the limit given by the (non-diversifiable) systemic risk. Finally, uncertainty regarding inter and intra correlation in such framework of credit portfolio models leads to a large and significant variation on the capital measure. Furthermore, the uncertainty implicit in the Basel II recommended bounds ($\rho_{BaselII} \in [0.12, 0.24]$) for asset correlation can lead to a very extreme underestimation of the capital measure (lower left section of the figure).

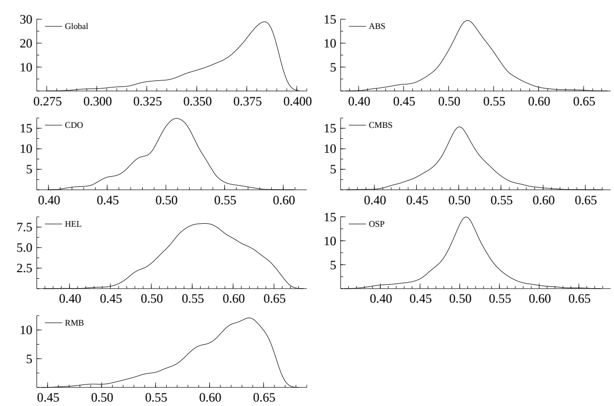
[14]The value $\Delta\rho_g = 0$ is included in the interval so as to nest the 1-F model in the 2-F specification.



(a) Economic Sectors



(b) World Regions



(c) Structured Products

Figure 4. Posteriors distributions for implied asset correlations (Model C).
Source: Author's estimation.

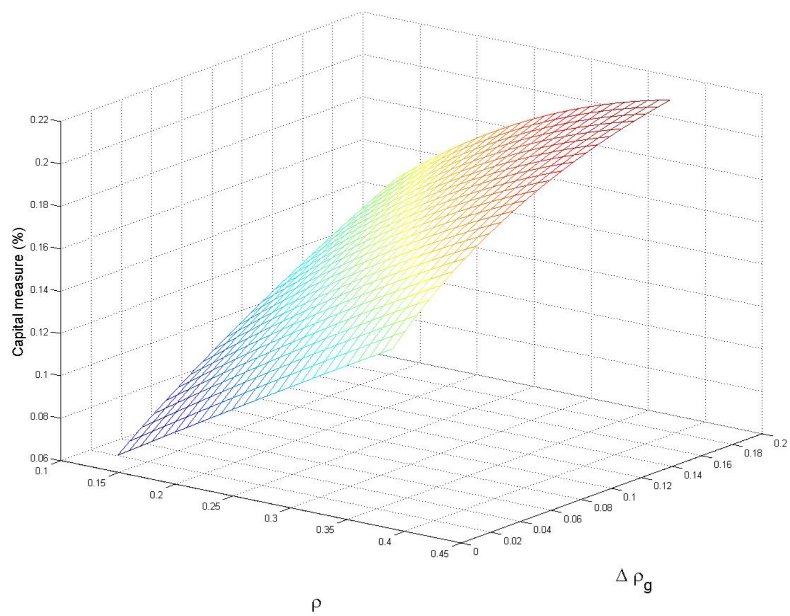


Figure 5. Implications on economic capital of the uncertainty over the estimate of asset correlations in a dynamic factor model of default risk.
Source: Author's compilation.

5 Attenuating over-dispersion in the data with a zero-inflated binomial model

The model in section 4 is affected by the large numbers of zeros across the time series of cross sectional units (zero-inflation). Banking, finance/insurance/real estate, sovereign/public finance, and utilities, have 26, 22, 30, 27 years (out of 40), respectively where there are no observed defaults. This excessive number zeros places a shadow of doubt over the properness of the binomial distribution as the right distribution for the default counts $y_{g,t}$. Following Hall (2000), we develop a model that explicitly accounts for a high frequency at zero by mixing discrete distributions with a degenerated distribution with point mass of one at zero.

Under the zero-inflated binomial (hereafter, ZIB) representation, the process generating the data has two states. A zero state from which only zero values are observed, and a binomial state from which all of the non-zero values and a few zero values are observed. The zero state has probability p_t. These assumptions have the following implications for the number of defaults:

$$y_{g,t}|\mathbf{B}_t \sim \begin{cases} 0, & p_t \\ Binomial(k_{g,t}, \pi_{g,t}), & 1 - p_t, \end{cases} \tag{8}$$

$$y_{g,t}|\mathbf{B}_t = \begin{cases} 0, & p_t + (1-p_t)(1-\pi_{g,t})^{k_{g,t}} \\ z_{g,t}, & (1-p_t)\begin{pmatrix} k_{g,t} \\ z_{g,t} \end{pmatrix} \pi_t^{z_{g,t}}(1-\pi_{g,t})^{k_{g,t}-z_{g,t}}, \end{cases} \tag{9}$$

where $z_{g,t}$ is the realization of the random variable $y_{g,t}$ at time t for group g. The probability of observing at least one default p_t depends on the total number of observed firms or products, $p_t = P(\tau k_{g,t})$. We select this particular functional form because of the direct link between the number of firms and the zero-inflation. In other words, the US data (since Moody's corporate and structured product data come predominantly from the US) shows no over-dispersion at all and it also represents a large part of the sample. Therefore the total number of observed firms or products is considered as a good predictor that there will be at least one default.

We apply the ZIB model to specification type C for the different groups of data in order to determine if there are significant gains in model fit of explicitly accounting for the over dispersion. As indicated by the information criteria there is some slight improvement in the fit. Furthermore, Table 8 presents a tally of the number of defaults predicted by each type of model, taking explicitly into account overestimation or underestimation of defaults in each time period. The last column indicates the total number of observed defaults per group. Results indicate that the ZIB improves on the other models for sectors, regions and products. The worst performing model is indeed the one factor approach (model A or B). Model C and the ZIB version of this same model provide and adequate fit to the default data (showing highest accuracy for regions and products), and they are able to capture the over-dispersion as well as the clustering of defaults observed in the corporate and the structured product data. Asset correlations derived from the ZIB model are presented in Table 9, they are slightly higher for sectors and regions and lower for products than those of model C (Table 6) but, the variation within each group is maintained. For example, as was observed in Section 4, the more volatile sectors and products are banking, energy, technology, and residential mortgage backed securities, respectively.

Table 8. Fitting the Corporate default data from 1970 to 2009 and Structured Product default data from 1982 to 2009.

	Model A AR	Model A *i.i.d.*	Model B AR	Model B *i.i.d.*	Model C	Model D	ZIB	Observed
Sectors	749	746	756	766	1077	987	1125	1379
BAN	4	3	1	3	51	53	53	63
CAI	283	283	280	280	309	317	324	370
COI	198	194	196	201	238	229	243	291
EAE	33	34	35	35	90	52	87	108
FIR	18	18	23	22	31	23	30	47
MED	15	14	15	15	26	16	29	46
RET	81	86	79	80	97	95	100	137
SOV	13	14	13	13	13	14	21	26
TEC	75	70	85	85	156	139	161	181
TRA	25	27	25	27	53	38	61	82
UTL	4	3	4	5	13	11	16	28
Regions	1104	1111	1101	1108	1205	1173	1210	1310
WEP	37	37	36	35	63	64	65	80
EEP	3	3	5	3	5	4	11	12
NOA	1013	1020	1004	1018	1049	1049	1045	1104
SCA	33	32	33	30	51	34	49	63
AOC	17	18	22	21	35	21	34	45
MDE	0	0	0	0	0	0	2	2
AFK	1	1	1	1	2	1	4	4
Products	1637	1637	2532	2517	3446	3442	3457	3486
ABS	129	128	1	2	67	68	69	75
CDO	219	219	355	346	966	964	968	975
CMBS	84	84	80	83	111	107	109	116
HEL	232	232	179	181	152	149	153	157
OSP	193	193	7	6	124	126	128	131
RMB	780	779	1909	1898	2026	2028	2030	2032

Note: The table presents the number of predicted defaults (at the relevant point in time) $(\widehat{y}_{g,t})$ by the different models as well as the total historical number of observed defaults (last column in the table) for the sectors, regions and structured products.
Source: Author's estimation.

Table 9. Asset correlations obtained from the default risk model. Zero-Inflated model (ZIB) of type C and Probit response function.

Model ZIB	2.5	Mean	97.5
Global	0.147	0.322	0.396
Banking	0.241	0.441	0.540
Capital industries	0.184	0.409	0.503
Consumer Industies	0.182	0.406	0.501
Energy & Environment	0.216	0.425	0.519
Finance, Insurance & Real Estate	0.184	0.409	0.506
Media & Publishing	0.182	0.411	0.510
Retail & Distribution	0.178	0.404	0.499
Sovereign & Public Finance	0.179	0.406	0.500
Technology	0.216	0.426	0.517
Transportation	0.176	0.404	0.500
Utilities	0.179	0.406	0.501
Global	0.065	0.242	0.391
Western Europe	0.088	0.312	0.500
Eastern Europe	0.093	0.328	0.525
North America	0.080	0.303	0.495
Central & South America	0.096	0.319	0.504
Asia & Oceania	0.096	0.323	0.508
Middle East	0.139	0.416	0.622
Africa	0.135	0.414	0.621
Global	0.280	0.347	0.390
ABS	0.405	0.499	0.597
CDO	0.397	0.481	0.556
CMBS	0.382	0.475	0.576
HEL	0.398	0.496	0.609
OSP	0.393	0.483	0.570
RMB	0.480	0.553	0.646

Note: The table presents the number of predicted defaults (at the relevant point in time) $(\widehat{y}_{g,t})$ by the different models as well as the total historical number of observed defaults (last column in the table) for the sectors, regions and structured products.

Source: Author's estimation.

6 Conclusions

An important change form the Basel I to the Basel II accord, with respect the technicalities associated to the estimation of capital requirements, addresses the issue of accounting for as much of the heterogeneity as possible in the determination of the risk weights associated with each position in a portfolio. This has been a topic of ongoing research. However, there are some issues so far unresolved: First, the professional tools that are available concentrate on equity data to measure dependence (correlation) across groups of issuers (industries, countries); even thought changes in equity data may not be an adequate proxy of the changes in credit quality. A good example of these methodologies is CreditMetrics and Moody's KMV.[15] Second, when rating data has been used to characterize dependence the authors (with some exceptions) have dealt with the problem of estimating the parameters of interest on aggregated data. In part this is due to the difficulties of working with the rating data where the rare events (defaults in particular) are the most interesting but making inference on them is complex.

This paper complements research on asset correlation estimates across sectors using rating data and presents some new results on regions and structured products.[16]. We use the methodology developed by Wendin (2006); Wendin and McNeil (2006) and McNeil and Wendin (2007), along with publicly available software WINBUGS. With the aggregate data and in the one factor framework, results are very similar to the ones obtained by the previous authors.

With Bayesian methods, we offer results for a large number of disaggregated units (11 sectors, 7 world regions and 6 structured products). Bayesian methods make it possible to obtain estimates even with over dispersion of the data (Zero-Inflation), they also allow for a straightforward set up of additional mixtures in order to properly account for the zeros through a Zero-Inflated Binomiam model (ZIP). This ZIP provides in some cases substantial improvement in fitting the time series of defaults.

The loading parameters of the factors across the sectors, regions and structured products models are in general statistically significant. From these factor weights it is possible to recover the asset correlations. Asset correlations are in most cases higher than the Basel II recommended values. They are also higher if the unobserved component is autoregressive as opposed to *i.i.d*. The two factor model with AR(1) dynamics for global factor and with a local systemic factor is able to reproduce better the observed number of defaults than the one factor framework recommended by Basel II. In the panels, the US data determines most of the global factor dynamics. This is expected since the database is predominately composed of US firms.

Overall this chapter presents a set of asset correlation estimates for economic sectors, world regions and structured products that can be used for credit portfolio modeling. It also indicates some caution in the use of the Basel

[15] See Gupton et al. (1997) and Crosbie (2005).

[16] See Gordy and Heitfield (2002); Demey et al. (2004); Servigny and Renault (2003); Wendin and McNeil (2006); McNeil and Wendin (2007)

recommended bounds for asset correlation in portfolio risk models. First, these bounds in general tend to be over optimistic with respect to the dependence structure that is consistent with the historical default data (figures 3 to 5). Second, with some certainty these bounds do not hold equally for all exposures (there are differences observed in the estimates of asset correlations between sectors, regions and structured products. Third, the modeling assumptions used in the estimation of these implied asset correlations carry substantial model risk to the measurement of economic capital. Therefore, a transparent and proper sensibility analysis to the assumptions that give rise to the dependence structure should be an integral part of the portfolio credit risk model validation process.

Appendix

A.1 Derivation of the full conditional distributions for Bayesian estimation.

We obtain Bayesian estimates of the parameters of interest through the use of a Markov Chain Monte Carlo algorithm such as the Gibbs sampler. Implementation of the Gibbs sampler requires the derivation of the full conditional distributions of the elements of the models. This set of full conditional distributions are sampled in a way so as to derive the joint posterior distributions of the parameters of interest. Borrowing notation from Gilks et al. (1996) and McNeil and Wendin (2007), $[X]$ denotes the (unconditional) density of X, $[X|Y]$ the conditional density of X given Y and $[X|.]$ the full conditional of X.

We derive in the following subsections the full conditional distributions for model A (univariate model), C (multivariate model) and the zero-inflated binomial model.

A.2 Model A (univariate one factor model)

In the univariate case $\underline{X} := (X_1, ..., X_T)$. First recall the main elements of this state space model: The measurement equation,

$$y_t|B_t \sim Binomial(k_t, \Phi\left(\mu - \frac{\sqrt{\rho}}{\sqrt{1-\rho}}B_t\right)), t = 1, .., T,$$

the state equation, $B_t = \psi B_{t-1} + \sqrt{1-\psi^2}\eta_t$, $\eta_t \sim N(0,1)$. With prior distributions for the unknown parameters $\psi \sim U(-1,1)$, $\varrho(\rho) := \frac{\sqrt{\rho}}{\sqrt{1-\rho}} \sim U(0,10)$ and $\mu \sim N(0, \sigma_\mu^2 = 10^3)$. Note that the unobserved component has a multivariate Gaussian distribution, $\underline{B} \sim N(\underline{0}, \Omega)$.

$$\Omega = \frac{1}{1-\psi^2}\begin{pmatrix} 1 & \psi & \dots & \psi^{T-1} \\ \psi & 1 & \ddots & \psi^{T-2} \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{T-1} & \psi^{T-2} & \dots & 1 \end{pmatrix}.$$

The multivariate Gaussian density of $\underline{B}$ is

$$f_{\underline{B}}(\psi) = (2\pi)^{-T/2} \mid \Omega^{-1} \mid^{1/2} exp\left[-\frac{1}{2}\left(\underline{B}'\Omega^{-1}\underline{B}\right)\right].$$

A first step in deriving the full conditional distribution is to write out the joint distribution function of the data and the unknowns (parameters and unobserved components). This joint distribution can be further simplified given the conditional independence and unconditional independence across some of it elements. For example, assumptions on the model determine that defaults (y_t) are independent across time conditional on the unobserved component B_t. Furthermore, the parameters of interest $(\psi, \varrho(\rho), \mu)$, are themselves independent

$$\begin{aligned}[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho),\psi] &= [\underline{y}\mid\underline{k},\underline{B},\mu,\varrho(\rho)][\underline{B}\mid\psi][\mu][\varrho(\rho)][\psi],\\ &= \left(\prod_{t=1}^{T}[y_t\mid k_t,B_t,\mu,\varrho(\rho)][B_t\mid\psi]\right)[\mu][\varrho(\rho)][\psi],\end{aligned}$$

this expression represents a form of fragmentation of the joint distribution of the data and the unknowns. In particular there are five relevant fragments (conditional and unconditional distributions). The final step of deriving the full conditional distribution of any of the parameters of interest is to apply the conditional probability formula and pick out only the fragments that depend explicitly on the parameter of interest. The sign $\propto$ denotes a form of equivalence (proportional to), during the process of selecting the relevant fragments.

The full conditional for ψ is:

$$\begin{aligned}[\psi\mid .] &= \frac{[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho),\psi]}{[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho)]}\propto[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho)]\propto[\underline{B}\mid\psi][\psi],\\ &\propto \mid\Omega^{-1}\mid^{1/2} exp\left[-\frac{1}{2}\left(\underline{B}'\Omega^{-1}\underline{B}\right)\right]\frac{1}{2}1_{(-1\le\psi\le1)}.\end{aligned}$$

An expression for the conditional distribution of defaults is required for all of the other full conditional distributions:

$$\prod_{t=1}^{T}[y_t\mid k_t,B_t,\mu,\varrho(\rho)]\propto\prod_{t=1}^{T}\Phi\left(\mu-\varrho(\rho)B_t\right)^{y_t}\left[1-\Phi\left(\mu-\varrho(\rho)B_t\right)\right]^{k_t-y_t}.$$

The full conditional for $\varrho(\rho)$ is:

$$\begin{aligned}[\varrho(\rho)\mid .] &= \frac{[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho),\psi]}{[\underline{y},\underline{k},\underline{B},\mu,\psi]}\propto[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho),\psi]\\ &\propto \prod_{t=1}^{T}[y_t\mid k_t,B_t,\mu,\varrho(\rho)][\varrho(\rho)],\\ &\propto \prod_{t=1}^{T}\Phi\left(\mu-\varrho(\rho)B_t\right)^{y_t}\left[1-\Phi\left(\mu-\varrho(\rho)B_t\right)\right]^{k_t-y_t}\frac{1}{10}1_{(0\le\varrho(\rho)\le10)}.\end{aligned}$$

The full conditional for μ is:

$$\begin{aligned}[\mu\mid .] &= \frac{[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho),\psi]}{[\underline{y},\underline{k},\underline{B},\varrho(\rho),\psi]}\propto[\underline{y},\underline{k},\underline{B},\mu,\varrho(\rho),\psi]\\ &\propto \prod_{t=1}^{T}[y_t\mid k_t,B_t,\mu,\varrho(\rho)][\mu],\\ &\propto \prod_{t=1}^{T}\Phi\left(\mu-\varrho(\rho)B_t\right)^{y_t}\left[1-\Phi\left(\mu-\varrho(\rho)B_t\right)\right]^{k_t-y_t}(2\pi\sigma_\mu^2)^{-1/2}\\ &\quad exp\left(-\frac{1}{2}\frac{\mu^2}{\sigma_\mu^2}\right).\end{aligned}$$

Since the unobserved component B_t is a T dimensional process then each component must be treated individually. It is also important to take into account that since B_t is AR(1) then the realization of such process depends on its immediate neighbors. Denote the vector $\underline{B}$ without the t element as $\underline{B}_{-t} := (B_1, \ldots, B_{t-1}, B_{t+1}.., B_T)$. Under these assumptions the full conditional for B_t is:

$$\begin{aligned}[B_t \mid .] &= \frac{[\underline{y}, \underline{k}, \underline{B}, \mu, \varrho(\rho), \psi]}{[\underline{y}, \underline{k}, \varrho(\rho), \psi]} \propto [\underline{y}, \underline{k}, \underline{B}, \mu, \varrho(\rho), \psi], \\ &\propto \prod_{t=1}^{T} [y_t \mid k_t, B_t, \mu, \varrho(\rho)][B_t \mid B_{t-1}, B_{t+1}, \psi].\end{aligned}$$

An smooth estimate (two-sided filter) is obtained for the conditional distribution of B_t given $\underline{B}_{-t}$, with a forward and backward looking element (one sided filter) at the origin and at the end, respectively,

$$[B_1 \mid B_1, B_2, \psi] = \left(2\pi \frac{1}{1-\psi^2}\right)^{-1/2} exp\left(-\frac{1}{2}\frac{(B_2 - \psi B_1)^2}{\frac{1}{1-\psi^2}}\right),$$

$$[B_t \mid B_{t-1}, B_{t+1}, \psi] = \left(2\pi \frac{1}{1-\psi^2}\right)^{-1/2} exp\left(-\frac{1}{2}\frac{(B_t - \frac{\psi}{2}(B_{t-1} + B_{t+1}))^2}{\frac{1}{1-\psi^2}}\right),$$

$$[B_T \mid B_{T-1}, B_T, \psi] = \left(2\pi \frac{1}{1-\psi^2}\right)^{-1/2} exp\left(-\frac{1}{2}\frac{(B_T - \psi B_{T-1})^2}{\frac{1}{1-\psi^2}}\right).$$

Because of the structure of the state-space model, it is not possible to reduced analytically the full conditional likelihood derived previously in order to get a closed-form distribution. The current setup requires the use of efficient algorithms (such as adaptive rejection sampling or Metropolis-Hastings) to sample the distributions, Gilks et al. (1996).

A.3 Model C (multivariate two factor model)

In the multivariate case $\underline{X} := (X_{1,1}, ..., X_{T,G})$. First recall the main elements of this state space model: The measurement equation,

$$y_{g,t}|\mathbf{B}_t \sim Binomial(k_{g,t}, \Phi(\mu_g - \varrho(\rho)B_t - \varrho(\rho)_g B_{t,g})), g = 1, .., G; t = 1, .., T,$$

the state equation, $\mathbf{B}_t = \Phi\mathbf{B}_{t-1} + \Theta\eta_t$, $\eta_t \sim N(0, I)$. $\Phi = diag(\psi, \psi_1, ..., \psi_G)$, $\Theta = diag(\sqrt{1-\psi^2}, \sqrt{1-\psi_1^2}, ..., \sqrt{1-\psi_G^2})$. With prior distributions for the unknown parameters $\psi, \psi_g \sim U(-1, 1)$, $\varrho(\rho) := \frac{\sqrt{\rho}}{\sqrt{1-\rho_g}}$,$\varrho_g(\rho) := \frac{\sqrt{\rho_g - \rho}}{\sqrt{1-\rho_g}} \sim U(0, 10)$ and $\mu_g \sim N(0, \sigma_\mu^2 = 10^3)$, $g = 1, ...G$. Note that the unobserved component has a multivariate Gaussian distribution, $\mathbf{B}_t \sim N(\Phi\mathbf{B}_{t-1}, \Omega)$. Then the multivariate Gaussian density of $\underline{B}$ is

$$f_{\mathbf{B}_t|\mathbf{B}_{t-1}}(\Phi) = (2\pi)^{-(G+1)/2} \mid \Omega^{-1} \mid^{1/2} exp\left[\left((\mathbf{B}_t - \Phi\mathbf{B}_{t-1})'\Omega^{-1}(\mathbf{B}_t - \Phi\mathbf{B}_{t-1})\right)\right].$$

The fragmentation of the joint distribution of the data and the unknowns, under this model, is:

$$\begin{aligned}[\underline{y}, \underline{k}, \underline{B}, \mu, \varrho(\rho), \psi] &= [\underline{y} \mid \underline{k}, \underline{B}, \mu, \varrho(\rho)][\underline{B} \mid \psi][\mu][\varrho(\rho)][\psi], \\ &= \left(\prod_{t=1}^{T}\prod_{g=1}^{G}[y_{g,t} \mid k_{g,t}, \mathbf{B}_t, \mu, \varrho(\rho)][\mathbf{B}_t \mid \psi]\right)[\mu][\varrho(\rho)][\psi],\end{aligned}$$

where $\mu := (\mu_1, ..., \mu_G)$,$\varrho(\rho) := (\varrho(\rho), \varrho(\rho)_1, ..., \varrho(\rho)_G)$, $\psi := (\psi, \psi_1, ..., \psi_G)$. The full conditional distributions of the elements in the multivariate models are found analogously, using the procedures presented for the univariate case.

A.4 Zero-Inflated Binomial model

In the multivariate case $\underline{X} := (X_{1,1}, ..., X_{T,G})$. First recall the main elements of this state space model that incorporates an additional mixture in order to distinguish the two relevant states $y_{g,t} = 0$ (no defaults) or $y_{g,t} \neq 0$ (in which case the number of defaults is denoted as $z_{g,t}$): The measurement equation capture these two states,

$$y_{g,t}|\mathbf{B}_t = \begin{cases} 0, & p_{g,t} + (1-p_t)(1-\pi_{g,t})^{k_{g,t}}, \\ z_{g,t}, & (1-p_{g,t})\begin{pmatrix}k_{g,t}\\ z_{g,t}\end{pmatrix}\pi_t^{z_{g,t}}(1-\pi_{g,t})^{k_{g,t}-z_{g,t}}, \end{cases}$$

the state equation, $\mathbf{B}_t = \Phi\mathbf{B}_{t-1} + \Theta\eta_t$, $\eta_t \sim N(0, I)$. The priors for $(\mu, \psi, \varrho(\rho))$ have the same characteristics of the previous model. An additional signal is introduced $p_{g,t} = P(\tau k_{g,t})$, with the following prior for $\tau \sim N(0, \sigma_\tau^2 = 10^3)$.

The easiest form of presenting the ZIB model is to segment the observed defaults $\underline{y} := (\underline{y}^0, \underline{z})$, where $\underline{z}$ denotes the non-zero values of $\underline{y}$, which is a stacked vector of of dimensions $((TxG)x1)$. Let $\underline{y}$ be arranged such that the first m elements are in $\underline{y}^0$ (zero defaults) and elements from $m+1$ to L are in $\underline{z}$ (non-zero defaults).

The fragmentation of the joint distribution of the data and the unknowns, under this model, is:

$$[\underline{y}, \underline{k}, \underline{B}, \underline{p}, \mu, \varrho(\rho), \psi, \tau] = [\underline{y} \mid \underline{k}, \underline{B}, \underline{p}, \mu, \varrho(\rho)][\underline{B} \mid \psi][\underline{p} \mid \underline{k}, \tau][\mu][\varrho(\rho)][\psi][\tau]$$

An expression for the conditional distribution of defaults is required for all of the other full conditional distributions. For observations $1...m$:

$$\begin{aligned}&\prod_{l=1}^{m}\prod_{g=1}^{G}[y_{g,l}^0 \mid k_{g,l}, \mathbf{B}_l, p_{g,l}, \mu, \varrho(\rho), \tau] \propto, \\ &\quad \prod_{l=1}^{m}\prod_{g=1}^{G} P(\tau k_{g,l}) + (1 - P(\tau k_{g,l}))[1 - \Phi(\mu_g - \varrho(\rho)B_l - \varrho(\rho)_g B_{l,g})]^{k_{g,l}},\end{aligned}$$

for observations $m+1...L$:

$$\prod_{l=m+1}^{L}\prod_{g=1}^{G}[z_{g,l} \mid k_{g,l}, \mathbf{B}_l, p_{g,l}, \mu, \varrho(\rho), \tau] \propto,$$

$$\prod_{l=m+1}^{L}\prod_{g=1}^{G}(1-P\left(\tau k_{g,l}\right))\Phi\left(\mu_g-\varrho(\rho)B_l-\varrho(\rho)_g B_{l,g}\right)^{z_{g,l}},$$

$$[1-\Phi\left(\mu_g-\varrho(\rho)B_l-\varrho(\rho)_g B_{l,g}\right)]^{k_{g,l}-z_{g,l}}.$$

The full conditional distributions of the elements in the ZIB model are found analogously, using the procedures presented for the univariate case.

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