OPTIMAL INCOME TAXATION WITH SINGLE AND COUPLE HOUSEHOLDS *

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ABSTRACT

This paper reviews the literature on optimal income taxation with single and couple households. In the seminal works of Mirrlees and Atkinson and Stiglitz the household is composed by one member. However, I describe a model where households can have more than one member. There is an economy which is composed by both one-member and two-member families. This structure introduces vertical and horizontal equity considerations. For linear taxation the results of Sheshinski hold, however in optimal income taxation additional results to those of Mirrlees should be considered.

Key words: optimal income taxation, linear taxation, vertical equity, horizontal equity.
JEL Classification: H21, H24.

RESUMEN

Este artículo hace una revisión de la literatura y describe un modelo general para entender el problema de la teoría óptima de impuestos cuando los hogares son compuestos de un solo miembro o dos miembros. En los trabajos iniciales de Mirrlees y Atkinson y Stiglitz los hogares estaban compuestos de un solo miembro. Sin embargo, la realidad presenta un situación distinta, yo describo un modelo donde la economía esta compuesta por hogares de un miembro o dos miembros. Esto introduce consideraciones de equidad horizontal y vertical. Para impuestos lineales los resultados del modelo de Sheshinski se mantienen; sin embargo, para el caso de impuestos óptimos al ingreso se deben tener en cuenta nuevos resultados.

Palabras clave: Impuestos óptimos, Impuestos lineales, equidad vertical, equidad horizontal.
Clasificación JEL: H21, H24.

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I. INTRODUCTION

This essay is about the treatment of single and couple families by the income tax schedule. The two seminal contributions on taxation of households are Mirrlees (1971) and Sheshinski (1972). One of the main assumptions of these papers is that households are either composed of one member or its members are homogeneous. This assumption has been used by most of the papers in the literature on taxation during the last three decades. However, it is natural to assume that households, in many cases, are composed by more than one member and that there may be some heterogeneity at the interior of the household.

The assumption that households' members are homogeneous makes it difficult to understand the diversity in the treatment of the family as a taxation unit around the world. Comparative studies across developed countries show big differences in the income tax schedules when family size is considered. In some countries single households face a higher tax burden than couples, in those countries joint income taxation for legally recognized couples is allowed. But one can also find countries that have a taxation system which is less biased towards couples and where couples face a higher tax burden than singles. The arguments that support each of this tax schedules emphasize one of three arguments. First, one can think that couples should pay fewer taxes than singles because they have more needs. Second, one can think that the lack of economies of scale in consumption in a single household calls for a favorable tax system for singles. Third, there is the issue of the feasibility of joint taxation, since the single-couple status may be difficult to verify and different taxation schemes may distort the decision to remain single or to live with a partner.

In the case of European countries there are diversities in the taxation of households[1]. One group of countries has both joint and individual tax schedules (i. e. France, Portugal and Luxembourg have mandatory joint taxation for couples for all incomes). In some cases couples would choose the regime with more financial benefits. However, in the last decade a general trend shows that several countries have been moving from joint taxation toward individual taxation (table 1 presents the tax system in European Union).

[1] An analysis of differences in tax systems in European countries and theirs relations with family topics is presented in O'Donoghue and Sutherland (1998).

Table 1
Income taxation in couples across European Union

Country	Type of Joint tax for couples	Joint tax*
Austria	None.	No
Belgium	Property income taxed jointly and family quotient used when one spouse has low income.	Yes
Denmark	Capital income taxed jointly.	No
Finland	None.	No
France	Family quotient.	Yes
Germany	Optional income splitting.	Yes
Greece	None.	Yes
Ireland	Optional income splitting.	Yes
Italy	None.	No
Luxembourg	Aggregation.	Yes
Netherlands	Property income is assessed with income of higher earner.	No
Portugal	Income splitting.	Yes
Spain	Optional aggregate taxation.	Yes
Sweden	None (joint taxation of wealth only)	No
UK	None.	No

* I keep the criteria of O'Donoghue and Sutherland (1998). A country that allows only joint taxation on capital income or self-employment income is not categorized in the group of countries with joint tax system. *Sources:* O'Donoghue and Sutherland (1998).

In the United States the discussion is also open. Joint taxation is allowed but a neutral treatment toward marriage status is at the core of the discussion. The government tries to understand what it is the effect of joint taxation on families' income. Under joint taxation households may face either a marriage tax bonus or a marriage tax penalty. The largest marriage tax bonus occurs when one spouse earns the totality of the income. The more evenly divided the income, the more likely a married couple will experience a marriage tax penalty. The largest tax penalties occur where the income is evenly divided between the two spouses (Esenwein, 2003).

A broad characterization of tax systems will distinguish between the three following systems. *Joint taxation*, in which the partners' incomes are added together and taxed at progressive marginal rates as if they had each earned one-half the income. *Individual taxation*, in which each partner's income is taxed separately, but according to the same progressive tax schedule. *Selective taxation*, in which secondary earners are taxed on a separate, lower, progressive tax schedule than that for primary earners.

In splitting income[2] both spouses face the same marginal tax rate. Equation (1) summarizes the aggregated tax function. The incomes of each member of the family the total income is taxed as if the unit were a single individual.

[2] In France, for example, the current income tax regime is an extension of the splitting system, which takes into account the presence of dependent children. In families without children the quotient method is equivalent to split taxation.

$$Tax = 2T\left(\frac{I_m + I_f}{2}\right) \tag{1}$$

For example, in France income is jointly taxed through a family quotient system. This process divides the family's taxable income by a quotient which depends on family size. The resulting calculated tax is multiplied by the same quotient. The quotient is increased by 1 or 0.5 for each extra child depending on the family structure. Table 2 shows the quotient family in France.

Table 2
Definition of the family quotient

Number of Children	Single	Married
0	1.0	2.0
1	2.0	2.5
2	2.5	3.0
3	3.5	4.0
4	4.5	5.0

Sources: O'Donoghue and Sutherland (1998).

Under independent taxation, the individuals' income is assessed separately. In equation (2) the tax schedule "*T*" is applied independently to the incomes of each partner in the household.

$$Tax = T(I_m) + T(I_f) \tag{2}$$

Selective taxation is given in equation (3). The tax schedule "*T*" is the sum of different tax function for each member in the household

$$Tax = T_m(I_m) + T_f(I_f) \tag{3}$$

In broader terms one can consider a general tax treatment for joint taxation $T(I_m, I_f)$ and ask the following questions. How must the government implement a redistributive tax system with a fair treatment for both single and couple households? In other words, how can the government achieve both horizontal equity and vertical equity principles in taxation with singles and couples? The first contribution that I was able to find that considers this problem was Balcer and Sadka (1986). They consider horizontal equity as an independent criterion to the maximization of welfare. These authors consider a government that maximizes a social welfare utility function subject to resources constraint, incentive compatibility constraint and horizontal equity constraint. However this approach is not totally satisfactory because the horizontal equity constraint might be very restrictive. In section 3 I discuss some examples and the recent contributions of Auerbach and Hassett (1999), which consider horizontal equity as a component of general aversion to inequality instead of a constraint in the maximization problem.

Second, how can the government redistribute in a setting where individuals differ in productivity and family size? This question has been considered by Boskin and Sheshinski (1983), Apps and Rees (1999) Gulg (2003), and Rees (2004) under the assumption of linear tax on income and by Schroyen (2003) and Cremer, Dellis and Pestieau (2003). In this essay my aim is to discuss contributions in which some of these questions have been already considered and the possible strands of research to answer these questions that have not been considered yet. I will focus on the contributions that analyze the tax treatment in singles and couples and how to design tax liabilities for households that have two labor supplies.

The content of this essay is the following: In section 2, I present the general structure of the problem. In section 3, I study the positive considerations about horizontal equity and general aversion to inequality. First, I discuss the effect of taking into account horizontal equity considerations as an independent concept (Balcer and Sadka, 1986). Second, I present the Auerbach and Hassett (1999) approach. In section 4, I discuss the normative problem of how to implement the tax system in an economy where the government maximizes an utilitarian welfare function without horizontal equity constraints. I present the contribution of Boskin and Sheshinski (1983) on linear tax income of couples and the extensions of Apps and Rees (1999) and Rees (2004). In the last section, I present the final considerations and the future agenda for research.

II. THE MODEL

The framework that I present in this essay is an extension of the standard optimal taxation model exposed by Mirrlees (1971) and Atkinson and Stiglitz (1972). Consider an economy consisting of a continuum of individuals who can be allocated into two different types of households. Households are composed by either one-member family (single) or two-member family (couple).[3] The number of households is finite an equal to η. The proportion of singles is η_s and the proportion of couples is η_c where $\eta = \eta_s + \eta_c$. Consequently, additionally to productivity[4] individuals differ in the size of the family to which they belong.

[3] In this work I analyze only households with one adult or two adults, children are not considered. Cremer, Dellis and Pestieau (2003) study an optimal income tax model where households differ in productivity and number of children. The government observes the number of children and before tax income. But it can not observe labor supply and productivity of parents. In this setting the traditional trade-offs between equity and efficiency hold (Sheshinski; 1972 and Atkinson and Stiglitz; 1972). The main contribution is that the marginal tax rates decrease with the family size in both linear and nonlinear taxes.

[4] This is the only source of heterogeneity in Mirrlees and in Atkinson and Stiglitz.

All citizens have preferences over pure private good c, private good with economies of scale h[5] and labor supply l, $u(c, h, l)$. I assume that $u(c, h, l)$ is strictly concave, twice continuously differentiable, strictly increasing in c and h, and strictly decreasing in l. Both the pure private good and the private good with economies of scale are normal goods that are produced with a linear technology.[6] Each citizen, regardless of his type, is endowed with one unit of time. In each type of household it is assumed that there is a distribution function of individuals' productivity. The productivity parameter of In singles is w_s, and it is distributed with a distribution function $f_s(w_s)$. In couples, the male's productivity, w_c^m, and female's productivity, w_c^f, parameters have a joint distribution function given by $f_c\left(w_c^m, w_c^f\right)$.

In the tradition of the optimal income tax literature, I assume that both labor supply and productivity are not observable by the government. However, the government can observe the before-tax income $I_s = w_s l_s$ for singles and both before-tax incomes $I_c^m = w_c^m l_c^m$ and $I_c^f = w_c^f l_c^f$ for couples. This rules out first-best taxation of types as a policy instrument while allowing nonlinear taxation incomes. In addition, the government observes the individual consumption in the pure private good and private good with economies of scale. Given this framework I discuss some additional aspects of the problem for each type of households.

1. Single Household

The standard analysis of Mirrlees (1971) suits this type of households. As before, there are η_s types of single households who differ in their productivity w_s, which is distributed with a distribution function $f_s(w_s)$. The informational structure is as before. In this type of households pure private good, c, and private good with economies of scale, H, are provided by the same person.

It is convenient to introduce the modified utility function:

$$v_s\left(c_s, h_s, I_s\right) \equiv u(c_s, h_s, I_s / w_s) \tag{4}$$

The budget constraint of the single is defined by:

$$c_s + h_s = w_s l_s - T_s(w_s l_s) \tag{5}$$

where $T_s(w_s l_s)$ is the tax function that the government charges to an individual with an income I. The individual's problem is:

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[5] In some cases good h can be considered as a pure 'public' household commodity good (see Schroyen; 2003) or a pure private household good (see Gulg; 2003).

$$\textbf{(P)}\quad \begin{array}{ll} \text{Max}_{c,h,I} & v_s\left(c_s, h_s, I_s\right) \\ \text{s.t.} & c_s + h_s = I_s - T(I_s) \end{array}$$

The first order conditions give us:

$$\frac{di}{dI} = -\frac{v_I'}{v_i'} = MRS_{i,Is} = 1 - T_s'(I_s) \text{ with } i = c, h \tag{6}$$

each individual solves the problem **(P).** The allocations $c(w_s)$ and $I_s(w_s)$ define the implicit tax structure $T_s(I_s(w_s))$. When the government wants to redistribute it should take into account the resources constraint and the informational problem. Formally, the government maximizes the utilitarian welfare function

$$\int v_s\left(c_s(w_s), h_s(w_s), {}^{I_s(w_s)}\!/_{w_s}\right) f_s(w_s) dw_s \tag{7}$$

Subject to the resource constraint

$$\int \left[T_s(I_s(w_s))\right] f_s(w_s) dw_s = 0 \tag{8}$$

and subject to the incentive compatibility constraints. In this problem there are two types of incentive compatibility constraints, global and local. The local incentive compatibility constraints imply the global ones. These constraints are given by the utility maximization of individuals:

$$\text{Max}_{\tilde{w}}\ v_s\left(c_s(\tilde{w}_s), h_s(\tilde{w}_s), I_s(\tilde{w}_s)/w_s\right) \tag{9}$$

where $\tilde{w}_s$ is the optimal response of a household with productivity w_s, the following first order condition is satisfied

$$\frac{\partial v_s}{\partial c_s}\frac{dc_s}{d\tilde{w}} + \frac{\partial v_s}{\partial h_s}\frac{dh_s}{d\tilde{w}} + \frac{1}{w}\frac{\partial v_s}{\partial l_s}\frac{dI_s}{d\tilde{w}} = 0 \tag{10}$$

The local incentive compatibility constraint states that

[6] In Apps and Rees (1999), Schroyen (2003) and Rees (2004) good *h* is a domestic good, produced by the household.

$$\left.\frac{dv_s}{dw}\right|_{\tilde{w}=w} = \frac{\partial v_s}{\partial c_s}\frac{dc_s}{d\tilde{w}} + \frac{\partial v_s}{\partial h_s}\frac{dh_s}{d\tilde{w}} + \frac{1}{w}\frac{\partial v_s}{\partial l_s}\frac{dI_s}{d\tilde{w}} - \frac{\partial v_s}{\partial l_s}\frac{I_s}{w_s^2} \tag{11}$$

Using the first order condition in (10), the local incentive compatibility constraint takes on the form that utility must increase with w_s

$$\frac{dv_s}{dw_s} = -\frac{\partial v_s}{\partial l_s}\frac{I_s}{w_s^2} \qquad (12)\,^{7}$$

Note that there are no new ingredients in the problem above (equations 7, 8 and 12), consequently there are no additional results in this framework and everything is well known about taxation with single households (see Mirrlees; 1972 and Atkinson and Stiglitz; 1972).

2. Couple households

In couple households the structure is more complex. Households differ not only in productivities but also in the number of members with different productivities. Consequently, the standard analysis of Mirrlees (1971) is not enough to explain the tax schedules faced by this type of households. As before, there are η_c households which differ, both in the male's productivity, w_c^m, and the female's productivity, w_c^f. These productivity parameters have a joint distribution function given by $f_c\left(w_c^m, w_c^f\right)$.

As in the singles' problem it is convenient to introduce the modified utility function of the individual

$$v_c^i\left(c_c^i, h_c, I_c^i\right) \equiv u(c_c^i, h_c, I_c^i / w_c^i) \qquad \forall\ i = m, f \tag{13}$$

The budget constraint is defined by

$$c_c^i + h_c = w_c^i l_c^i + \alpha_c^j h_c - T_c(w_c^m l_c^m, w_c^f l_c^f) \quad \forall\ i \neq j, \text{ and } i, j = m, f \tag{14}$$

Note the new ingredients in the individual's problem for couple households. First the tax function depends not only on the own individual's income but also on the partner's income. As before under joint taxation $T_c(.,.)$ is a function of both $w_c^m l_c^m$ and $w_c^f l_c^f$. However if households face an individual tax system then the tax function would be defined by $T(w_c^i l_c^i)$ for all $i = m, f$. Under joint taxation the tax function is not separable,

[7] In order to be available to solve this problem additional assumptions over the distribution function are required.

meaning that the tax paid by one member of the couple depends on the income of the other member.

Second, members of a couple can share the consumption of some goods (which cannot be done by singles). Each individual chooses the optimal levels of pure private good, private good with economies of scale and labor supply subject to the budget constraint (14). One unit of good *h* is consumed by both members in the household.[8] The term $\alpha_c^j h_c$ in (14) is the member *j's* contribution to the good *h*. The sum of the contribution "parameters" is equal to one. $\Sigma_{k=i,j}\alpha_c^k = 1$ Note that if $\alpha_c^j = 0$ (i.e. member *j's* contribution is zero) individual *i* has to buy the total quantity of good *h*, however if $\alpha_c^j = 1$ the individual *i's* pure private consumption is equal to the after tax income

$$w_c^i l_c^i - T_c(w_c^m l_c^m, w_c^f l_c^f)$$

In general, α_c^j is not a parameter, it may depend on the male's and female's productivity parameters $\alpha_c^i(w_c^m, w_c^f)\ \ \forall\ \ i = m, f$.

The individual's problem in the couple case is not standard as it is for the case of singles. In addition each member should decide the optimal level of the good *h* and his contribution for financing it. The standard theory does not apply here and the problem requires contributions about family decisions literature.

There are several theories that have studied this intra-family decision problem. The application of one of them implies a different structure to the problem, which may determine how the individuals choose consumption, labor supply, level of public good and the contributions for financing this public good. These choices imply what is the information structure of the problem and the form of the incentive compatibility constraint.

One approach is the dictatorship solution, which was developed by Becker (1981). In this case one member chooses allocation of labor supplies, private consumptions, and level of public good and contributions of each member for financing it. The unit of decision is the household which has the same utility as the "dictator" member. To consider the well-being of other members in the household, this approach may assume that the "dictator" has an altruism term in the utility function. For example,

$$v_c^i\left(c_c^i, h_c, I_c^i, c_c^j,\right) \equiv u(c_c^i, h_c, I_c^i / w_c^i) + \delta\mu(c_c^j) \quad \forall\ i \neq j \text{ and } i, j = m, f \tag{15}$$

8 In this way good *h* is consider a "public good" as in Schroyen (2003).

This term shows the level of caring that the household has on the well-being of those members that do not participate in the household decisions (term δ in equation 15). Under this approach the information structure is determined by the utilities possibilities that face the dictator member.

Apps and Rees (1999), Schroyen (2003) and Rees (2004) have mixed the household production theory with the hypothesis of Pareto efficiency in the household decision problem, introduced by Samuelson (1956). Household members behave cooperatively so all possibilities of costless welfare improvement are exhausted. The mechanism to solve this is the following. Each member has a reservation utility level that has to be weakly exceeded for each member to remain within the household. The authors assume that this constraint is not binding at the equilibrium.[9] The household maximizes the utility of one member subject to the participation constraint of another member and the budget constraint (I explain the household production model in Annex 1). These authors do not model any specific allocation rule. However, they suggest that their models naturally allow the analysis of intra-family distributional decisions. I discuss below the contribution of Schroyen (2003) about how this approach determines the incentive compatibility constraints that may be faced by the household.

Gulg (2003) expands the Apps and Rees's (1999) model by specifying how a couple shares resources. Gulg (2003) uses two approaches to explain how the spouses share the household decisions. First, she applies the family bargaining model. In this model a crucial point is that the individuals' income determines the bargaining power. The equilibrium concept is the Nash Bargaining Solution.[10] Second, the competitive approach to intra-family distribution, which is similar to the Chiappori's (1988) collective model. This approach requires that the family is on the Pareto frontier of the utility possibilities set which is achieved by the mechanism explained in Apps and Rees (1999) above.

If the economy is populated only by couples and assuming a specific theory to solve the individual's problem it would be possible to apply the indirect approach to derive the couple's tax function. The problem of the government is to maximize the utilitarian welfare function

$$\begin{aligned}&\iint\left[v_c^m\left(c^m(w_c^m,w_c^f),h_c(w_c^m,w_c^f),I^m(w_c^m,w_c^f)/w_c^m\right)\right]f_c(w_c^m,w_c^f)dw_c^m dw_c^f+\\&\iint\left[v_c^f\left(c^f(w_c^m,w_c^f),h_c(w_c^m,w_c^f),I^f(w_c^m,w_c^f)/w_c^f\right)\right]f_c(w_c^m,w_c^f)dw_c^m dw_c^f\end{aligned} \tag{16}$$

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9 With this assumption Apps and Rees (1999) and Schroyen (2003) do not explain how the tax policy parameters in the individual utility function may affect the incentives to create or destroy a household. In other words, if to live as a couple is consider as a coalition the literature does not explain how this coalition is created or destroyed.

10 It is important to say that there is an emerging literature on non-coopertaive bargaining in marriage (see Lundberg and Pollak, 1996), which will be consider for future research.

Subject to the resource constraint

$$\iint \left[2T_c(I_c^m(w_c^m, w_c^f), I_c^f(w_c^m, w_c^f) \right] f_c(w_c^m, w_c^f) dw_c^m dw_c^f = 0 \qquad (17)$$

and subject to the incentive compatibility constraints. Here the condition explained in equation (12) is not sufficient. In the case of the single it is only required that the utility is increasing in w_s. However, in couples the utility depends of both w_c^m and w_c^f, which affect the allocation decisions of each member within the household. As was stated before each family decision theory may put a different informational structure on the problem.

In this literature these incentive compatibility constraints have been treated under particular simplifying assumptions. Both Boskin and Sheshinski (1983) and Apps and Rees (1999) solve the problem under optimal linear taxation, consequently, they do not need to introduce an incentive compatibility constraint in the problem. Balcer and Sadka (1986) assume that in each household there is only one earner-wage member and the government can observe if one member belongs to a couple or single household. In this sense the incentive compatibility constraint is standard as in equation (12), for each type of household. However, in Schroyen (2003) the incentive compatibility constraint depends on the trading possibilities within the household, in other words how the utility of each member can be affected by the possibility to trade with each member in the household. I discuss in more detail this argument.

In Schroyen the incentive compatibility depends on the tax arbitrage possibilities that an individual tax system may offer to the household. Within the household, its members can exchange leisure for consumption by rescheduling duties in housework and compensating each other for this. Consequently, when the government designs the tax schedule, it should take into account that people may engage in this kind of arbitrage.

In an economy where there are two types of productivities the government designs a compatible tax system, in which a high-productivity person prefers high income and pays the corresponding tax. In addition assume that two high-ability persons form a couple household. The trade arbitrage allows them to agree that one of them takes a part-time job on the labor market, earning a low income level with low tax liability and the other earns the high income. The government should consider these incentives in the tax system.

The specific form of the incentive compatibility constraints in Schroyen's (2003) work are determined by two conditions. First, if two household members choose the same bundle of consumption and income,[11] the incentive compatibility constraint is

[11] Schroyen calls this case the uniform labor market behavior.

the standard. It means that the problem of couples under this assumption can be solved as in Mirrlees (1971). Second, two members do not choose the same bundle and this behavior generates a trade possibilities, which may imply Pareto improvements within the household. The author calls this the Lower Envelope condition (LE). If the LE condition is satisfied then the government should consider the incentives of a household composed by two high- productivity individuals to pretend that one of its members is low-ability in order to get informational rents because of the trading possibilities.

As stated before, the possibilities to solve the government problem under couples depends on the theory for family that is used. The most relevant contributions that I was able to find in the literature solve specific cases of the general problem. Boskin and Sheshinski (1983) are the first to investigate the question of whether the individual or the household is the appropriate tax unit. They use a linear tax function in an economy that is populated only by couples. In their model a private good with economies of scale is not considered.

Apps and Rees (1999) extend Boskin and Sheshinski's model and analyze the problem under household production theory. Additional works in the same vein have been developed by Schroyen (2003) and Gulg (2003). In Schroyen (2003) the most relevant contribution is that the government should design an incentive compatibility tax system. These incentive compatibility constraints are more complex that those in the standard model, since the member within a household faces trading possibilities.

3. Mixed economy (single and couple households)

The mixed problem is interesting for two reasons. First, if the economy is populated by both singles and couples households the government must consider inequality between members of a same group and across groups. Second, the household is not necessarily an *a priori* verifiable coalition (Schroyen; 2003). It means that in some cases the government is not able to screen between singles and couples.[12] Under these arguments the problem of the government is to maximize the individuals' utility function

$$\begin{gathered}\int\left[v_s\left(x(w_s), I_s(w_s)/w_s\right)\right]f_s(w_s)dw_s + \\ \iint\left[v_c^m\left(c^m(w_c^m, w_c^f), h_c(w_c^m, w_c^f), I^m(w_c^m, w_c^f)/w_c^m\right)\right]f_c(w_c^m, w_c^f)dw_c^m dw_c^f + \\ \iint\left[v_c^f\left(c^f(w_c^m, w_c^f), h_c(w_c^m, w_c^f), I^f(w_c^m, w_c^f)/w_c^f\right)\right]f_c(w_c^m, w_c^f)dw_c^m dw_c^f\end{gathered} \tag{18}$$

[12] In section 3 I discuss an example where the government is not able to observe if each individual belongs to a single or a couple household. It seems a strong assumption, but consider a case where two individual were living as a married couple for several years. Currently, they do not live together, but they do not report it in order to receive the tax benefits. Empirically in USA the divorce rates increase when the tax benefits for couples decrease (See Alm and Whittington; 1997, Gelardi 1996).

Subject to the resource constraint

$$\int\left[T_s(I_s(w_s))\right]f_s(w_s)dw_s+\iint\left[2T_c(I_c^m(w_c^m,w_c^f),I_c^f(w_c^m,w_c^f)\right]f_c(w_c^m,w_c^f)dw_c^m dw_c^f=0 \quad (19)$$

and subject to the incentive compatibility constraints of both single and couples individuals. Again the set of incentive compatibility constraints depends on a specific family decision theory. In addition to the previous discussion the government needs to consider the possibility that a single mimics a couple.

As before there is not a complete understanding in the literature of this general problem. Balcer and Sadka (1986) solve a problem where singles (small families) and couples (large families) can be considered in the design of a tax system. However, those authors use "equivalences scale methodology" in order to introduce the family size effect. Under this assumption they transform the problem in one dimension type. Consequently, the standard results hold in their setting.

III. HORIZONTAL VERSUS VERTICAL EQUITY

The principle of horizontal equity states that those who are in all relevant senses identical should be treated identically (Atkinson and Stiglitz; 1980). The implementation of this principle requires a definition of both "relevant" and "treated identically" definitions. For example in an economy that is populated by two households with the same income but with different family size (i.e. if $w_s l_s = w_c^m l_c^m + w_c^f l_c^f$) each member should achieve the same utility level. The principle of vertical equity concerns the treatment of people who are unequal, meaning redistribution from poor to rich (Atkinson and Stiglitz; 1980).

Under these definitions the government's problem, in the previous section, shows conflicts in the use of both the horizontal equity and vertical equity principle. These conflicts arise because the government wants to redistribute from rich to poor but holding a fair treatment of both single and couple households. In this context the role of horizontal equity in the design of any tax system has been considered as an important goal. However, an adequate definition of this concept has been elusive while other definitions of inequality, as vertical equity, are well-accepted (Auerbach and Hassett; 1999).

The question in the positive domain is how to redistribute when the economy is populated by single and couple households. As stated in the introduction, two arguments call for a bias either for singles or for couples in the design of tax schedules. Those arguments imply that the tax system has to consider not only vertical equity but also horizontal equity criteria. The implementation of one of them may exclude the possibility to use another one. Both arguments can be explained in the context of the general structure of section 2, assuming two simple cases.

In the first case I assume there is an economy with two types of households, singles and couples. Without loss of generality I assume that in couples only males have a labor income $I_c^m = w_c^m l_c^m$.[13] The budget constraint for single families is given by equation (5)

$$c_s + h_s = w_s l_s - T_s(w_s l_s) \tag{5}$$

And the budget constraint for couple households is given by

$$c_c^m + c_c^f + h_c = w_c^m l_c^m - T_c(w_c^m l_c^m) \tag{20}$$

In this case government implements both a progressive tax schedule and a tax schedule that equalizes the utility of members in couples with that of singles. Consequently for the same level of productivity, $w_s = w_c^m$, the government should tax each household in order to equalize the utilities of individuals in each type of households. Formally:

$$v_s\left(c_c(w_s), h_s(w_s), {}^{I_s(w_s)}\!/_{w_s}\right) = v_c\left(c_c^m(w_c^m), h_c(w_c^m), {}^{I_c^m(w_c^m)}\!/_{w_c^m}\right) + \delta v_c\left(c_c^f(w_c^m), h_c(w_c^m)\right) \tag{21}$$

Horizontal equity calls for a favorable tax treatment of couples because they face higher needs than singles. Balcer and Sadka (1986) apply horizontal equity principle in this way (see the next section).

The second case illustrates the argument for a biased treatment for singles. I assume that the government's problem follows the previous section. In addition I consider a case where the male's and female's productivities are the same and the contributions for good *h* are one half for each member $\alpha_c^i = 1/2 \quad \forall \quad i = m, f$.[14] I assume that for the same level of productivities, $w_s = w_c^i$, across types of households the level of good *h* is also the same. The budget constraint for singles is given by equation (5). Now the budget constraint for each member in a couple is given by

$$c_c^i + 1/2\, h_c = w_c^i l_c^i - T_c(w_c^m l_c^m, w_c^f l_c^f) \quad \forall \; i = m, f \tag{22}$$

13 In this case it is necessary to assume that males care about the welfare of females. This can be introduced as an altruism term in the male's utility function (see equation 15, above).

14 This assumption is not critical but it allows me to simplify the problem. With this assumption I can compare male and female with a single without additional considerations.

From this constraint and the assumptions above it is easy to see that without taxes couples are better off than singles in terms of private consumption, meaning that $c_s < c_c^i \quad \forall \quad i = m, f$.

$$
\begin{aligned}
& w_c^i l_c^i = w_s l_s \quad , \quad h_c = h_s \quad \text{and} \quad T(\bullet) = 0 \\
& \Rightarrow \\
& c_c^i + 1/2\, h_c = w_c^i l_c^i \quad \text{and} \quad c_s + h_s = w_s l_s \\
& \Rightarrow \\
& c_s = c_c^i - 1/2\, h_c \quad \Rightarrow \quad c_s < c_c^i
\end{aligned}
\tag{23}
$$

Horizontal equity implies that the government should tax each household in order to equalize the utilities of individuals in each type of household. Formally this means that

$$
\begin{aligned}
& v_s\left(c_s(w_s), h_s(w_s), {I_s(w_s)}\big/{w_s} \right) = v_c\left(c_c^m(w_c^m, w_c^f), h_c(w_c^m, w_c^f), {I_c^m(w_c^m, w_c^f)}\big/{w_c^m} \right) = \\
& v_c\left(c_c^f(w_c^m, w_c^f), h_c(w_c^m, w_c^f), {I_c^f(w_c^m, w_c^f)}\big/{w_c^f} \right)
\end{aligned}
\tag{24}
$$

Horizontal equity calls for a favorable tax treatment of singles because of economies of scale in the consumption of good *h*.

In the next section I present the evolution of these two approaches. First, horizontal equity has been considered as an independent concept of a maximization of a welfare utility function. Consequently, it is introduced in the problem as an additional constraint. With this definition two problems arise: how to justify this criterion and how to define the group of individuals when the border between an individual and other is not clear. Second, instead of imposing horizontal equity as an independent criterion I will consider the discussion where it is a component of general aversion to inequality, following Auerbach and Hassett (1999).

1. Horizontal equity as independent criteria of general aversion to inequality

Balcer and Sadka (1986) explain how the differences in family size should be treated by the optimal income tax system, in order to achieve *horizontal equity*. They use equivalences scale methodology to compare the utilities between two types of households.[15] The model considers households with one-member and households

[15] A Equivalences scale allows comparing consumption levels between households. In the model explained in section 2 private good with economies of scale replaces the scaling methodology.

with two-members. The “standard adults” in the one-member family is equal to 1 and in the two-member family is equal to *z.* Each household chooses consumption *x* and labor supply *l*. Consequently with the equivalence scale methodology the preferences of two-members over bundles *(x,l)* are the same as the one-member family’s preferences over (*x/z,l*). Additionally they assume that each family has one wage earner. Note that the problem in Balcer and Sadka’s model is equivalent to the first case explained in the previous section.

In Balcer and Sadka horizontal equity is not derived from a general aversion to inequality. Consequently horizontal equity is not an element in the objective function of government welfare maximization; rather, it is a constraint that the government has to achieve. Following Atkinson and Stiglitz (1980) the horizontal equity principle implies that families with different sizes should attain the same level of utility. In this setting the government maximizes a utilitarian social welfare function subject to horizontal equity constraint and resources constraint. Formally, the problem is the following

$$\text{Max } v_s\left(x_s, l_s\right) + v_c^a\left(x_c^a / z, l_c^m\right) \tag{25}$$

$$\text{s.t. } v_s\left(x_s, l_s\right) = v_c^a\left(x_c^a / z, l_c^m\right) \tag{26}$$

$$x_s + x_c^a + R \leq w_s l_s + w_c^m l_c^m \tag{27}$$

where *R* is the government revenue. $v_s(.,.)$, x_s, l_s, and l_c^m are defined as before. $v_c^a(.,.)$ is the aggregated utility function and $x_c^a = (c_c^m, c_c^f, h_c)$ aggregated consumption of couples. I use this structure instead of that in section 2 to follow Balcer and Sadka. However, the structure introduced in section 2 differs in several ways from that in Balcer and Sadka. First, in the problem of section 2 each individual in the household chooses the consumption goods allocation, contribution for good *h* and labor supply, but in Balcer and Sadka’s model households are the unit of decision. Second, in my structure the members within the household differ in productivities, but in Balcer and Sadka $w_c^m = w_c^f$. Third, the good with economies of scale *h* is replaced by the equivalence scale methodology. Finally, the pure private consumption good is equal for both male and female, it means that $c_c^m = c_c^f$. Consequently, the problem underlines in section 2 is more general that Balcer and Sadka’s model, and would allow to answer questions for which Balcer and Sadka’s setting is inappropriate.

When lump-sum tax and transfers are allowed and the horizontal equity constraint is not binding (i. e. equation 26 does not matter) the standard problem maximizes (25) with respect to (27). The first order conditions imply that:

$$MRS^{s}_{x,l} = \frac{u_l\left(x_s, l_s\right)}{u_x\left(x_s, l_s\right)} = -w_s \tag{28}$$

$$MRS^{c}_{x,l} = \frac{zu_l\left(x^a_c / z, l^m_c\right)}{u_x\left(x^a_c / z, l^m_c\right)} = -w^m_c \tag{29}$$

If the families have the same productivities $w^m_c = w_s = w$, the Pareto efficiency condition requires that

$$MRS^{s}_{x,l} = MRS^{c}_{x,l} = -w \tag{30}$$

Because $z > 1$, the last condition rules out the possibility of achieving horizontal equity by equalizing labor supplies and per-standard-adult consumptions (i.e. $l_s = l^m_c$ and $x_s = x^a_c / z$). If $v(\cdot)$ is strictly concave, couples should work and consume (per-standard adult) less than the singles. In terms of efficiency for society, 1 euro in hands of a couple implies consumption per-standard-adult of $1/z$ and for a single household means a consumption of 1 euro. In this sense single households are more efficient in consumption. Moreover, in order to get horizontal equity single households need to work harder. Balcer and Sadka's model shows that under lump-sum tax horizontal equity implies more consumption and more labor supply for single households.

However if lump-sum taxes and transfers are not allowed the only way to get horizontal equity is when $l_s = l^m_c$ and $x_s = x^a_c / z$. No matter what the social welfare function is; there is only one way to achieve horizontal equity: all families with the same earning ability must provide the same supply of labor and have the same per-standard adult consumption, regardless of family size. This result implies that the group of one member households faces higher marginal tax rates than the group of two-member households.

Under optimal tax income the government maximizes an utilitarian social welfare function.

$$\max \int v_s\left[x_s(w), l_s(w)\right] f_s(w)dw + \int v^a_c \left[x^a_c(w) \big/ z, l^m_c(w)\right] f_c(w)dw \tag{31}$$

Subject to resource constraint

$$R \leq \left[\int \left(wl_s(w) - x_s(w)\right) f_s(w)dw\right] + \left[\int \left(wl^m_c(w) - x^a_c(w)\right) f_c(w)dw\right] \tag{32}$$

The incentive compatibility constraint

$$\frac{dv_i}{dw} = -\frac{\partial v_i}{\partial l_i}\frac{I_i}{w^2} \quad \forall \quad i = s, c \tag{33}$$

And the horizontal equity constraint

$$v_s\left[x_s(w), l_s(w)\right] = v_c^a\left[x_c^a(w)\big/z, l_c^m(w)\right] \quad \forall w \tag{34}$$

Balcer and Sadka show that when horizontal equity must be achieved by the income tax, the per-standard adult consumption and the labor supplies of households must be equals $l_s(w) = l_c(w)$ and $x_s(w) = x_c(w)/z$ for all w. Moreover, the tax rates are related to each other by

$$T_s(I) = T_c(I)/z + I(z-1)/z \tag{35}$$

The marginal tax rates are given by

$$\begin{aligned} &T_s'(I) = T_c'(I)\big/z + (z-1)/z \\ &\Rightarrow \\ &T_c'(I) = zT_s'(I) - z + 1 \\ &\Rightarrow \\ &T_c'(I) < T_s'(I) \end{aligned} \tag{36}$$

The group of one-member households faces higher marginal tax rates. This is coherent with the problem because if horizontal equity requires the same labor supply, the government must distort the labor supply of the one member household. However, the possibilities to achieve horizontal equity would be undermined by information problems.

This result is the consequence of two important things. First, horizontal equity is an independent component of the utilitarian welfare function. Consequently, the only way to achieve it is to impose an additional constraint in the maximization problem that reduces the government's degree of freedom. Second, in Balcer and Sadka (1986) individuals differ in productivity and family size. However, they introduce the effect of family size taking into account the differences in the level of consumption using equivalences of scale. Consequently, Mirrlees' results hold since the set of types has one dimension.

Finally, Balcer and Sadka's (1986) work finds that income taxation with horizontal equity causes some additional deadweight loss.[16] It is a direct consequence of the constraint (26) in the government's problem. The main question here is how this deadweight loss will change when a new concept for horizontal equity is introduced.

2. Horizontal equity as a component of general aversion to inequality

In Balcer and Sadka (1986) horizontal equity is introduced as a constraint in the maximization problem of the government. This requires an adequate definition for this principle. Following Atkinson and Stiglitz (1980) the principle requires households with the same level of income to achieve the same level of utility. In Balcer and Sadka's model this implies a favorable tax function for large families. However, the informational structure may undermine the possibilities to achieve horizontal equity.

For example assume an hypothetical case where all individuals are equal in productivities. Redistribution from rich to poor does not matter. However, the individuals differ in the fact that they will live alone or with a partner. They have the possibility to live in couple or to remain single, but whether they live in couple is a random phenomenon. The question refers to the possibilities to achieve horizontal equity in the context of Balcer and Sadka. The answer depends of the informational structure. In the section 3.2.2 I discuss a new approach about horizontal equity as a general component for inequality.

2.1 A new measure of horizontal equity

In this section I present the contribution of Auerbach and Hassett (1999), whose aim is to propose a new way to measure horizontal equity. In Auerbach and Hassett (1999) horizontal equity is a component of a general evaluation of inequality. They consider a problem where households differ in income (global differences) and if they are a couple or single household (local differences).[17] The contribution of Auerbach and Hassett (1999) states that if the preferences to avoid horizontal inequality and vertical inequality are flexible, which means that it does not put the same weight to aversion to inequality within income class and across the income classes, it is possible to decompose the general aversion in two indexes (i.e. vertical and horizontal index). For the rest of this section I follow Auerbach and Hassett's (1999) work.

In Auerbach and Hassett (1999) the concept of horizontal equity is based on the idea that there are M classes of households. Each class has the characteristic that all households have the same level of income. Within each class there are a finite number

[16] They found that horizontal equity deadweight loss income taxation is not significant in some simulations.

[17] In this section I discuss the contributions in a setting where the unit of decision is the household. It can be generalized for individuals.

of households N_1 which can be a single or couple household. Where there are n_i^c couples and n_i^s singles with $N_i = n_i^c + n_i^s$. As in Atkinson (1970) it is possible to define a function based on individual after tax incomes that exhibits an aversion to inequality, among different levels of after-tax income:[18]

$$W = \left[\sum_{i=1}^{M} \sum_{j=s,c} (n_i^j (I_i - T_{ij}))^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{37}$$

where I_i is the before-tax income of households in group *i*, and T_{ij} is the tax function for a household of type *j* in the group *i.* Note that γ is the parameter that captures the aversion to inequality. It is a measure of the relative sensitivity to transfers at different income levels. If γ is high then we have more weight to transfers at the lower rather than at the top of the distribution, in other words society is more averse to inequality across classes.

Expression (36) has two important properties. First, it respects the Pareto principle, in that it is increasing in each household's after-tax income $\omega_{ij} = I_i - T_{ij}$.

$$\frac{\partial W}{\partial \omega_{ij}} = \frac{1}{1-\gamma} \left[\sum_{i=1}^{M} \sum_{j=1}^{N_i} (I_i - T_{ij})^{1-\gamma} \right]^{\frac{1}{1-\gamma}-1} (1-\gamma)\omega_{ij}^{-\lambda} > 0 \tag{38}$$

Second, it is increasing with respect to any change from an individual with higher after-tax income to one with lower after-tax income. Thus, it simultaneously incorporates notions of vertical equity and of horizontal equity.

A relevant weakness in function (37) is that it constrains differences among households within any class *i* to induce the same loss of social welfare as differences among households. But if horizontal equity is an independent concept, it seems both necessary and appropriate to distinguish between these two types of inequality. Large differences among similar households might be viewed as intrinsically arbitrary or they might be viewed as more costly because households compare themselves to those with similar characteristics. This suggests replacing (37) with (39)

$$W = \left[\sum_{i=1}^{M} N_i \left(\frac{1}{N_i} \sum_{j=s,c} (n_i^j (I_i - T_{ij}))^{1-\gamma} \right)^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-p}} \tag{39}$$

[18] Auerbach and Hassett (1999) avoid the problem about how to design the tax rate for individuals by assuming that after-tax income is given.

where n_i^c is a measure of inequality aversion within a group and ρ is a measure of inequality across groups. Note that in equation (39) function W still respects the Pareto principle, however it does not satisfy the principle that a comparison of any two outcomes should depend only on the well-being of households who are not indifferent to the outcomes. For example, there are two income classes, one single and one couple household in each class, and $\gamma > 0$ and $\rho = 0$. Equation (39) is given by

$$W = ((I_1 - T_{1s})^{1-\gamma} + (I_1 - T_{1c})^{1-\gamma})^{\frac{1}{1-\gamma}} + ((I_2 - T_{2s})^{1-\gamma} + (I_2 - T_{2c})^{1-\gamma})^{\frac{1}{1-\gamma}} \quad (40)$$

	Single	Couple
Income 1	I_{1s}	I_{1c}
Income 2	I_{2s}	I_{2c}

Further, assume that $I_1 = I_2$, and there are two outcomes (A and B) where the tax liability of couples does not change. This means that $\bar{T}_{1c}$ and $\bar{T}_{2c}$ are fixed. Suppose that under outcome A $T_{1s} = \varepsilon > 0$ and $T_{2s} = 0$, while under outcome B it is the contrary, $T_{1s} = 0$ and $T_{2s} = \varepsilon > 0$.

Outcome A

	Single	Couple
Income 1	$T_{1s} = \varepsilon > 0$	$\bar{T}_{1c}$
Income 2	$T_{2s} = 0$	$\bar{T}_{2c}$

Outcome B

	Single	Couple
Income 1	$T_{1s} = 0$	$\bar{T}_{1c}$
Income 2	$T_{2s} = \varepsilon > 0$	$\bar{T}_{2c}$

If $\bar{T}_{1c}$ and $\bar{T}_{2c}$ are both different to zero and ε, both outcomes generate horizontal inequality. The government has two options for the tax system. The first option modifies the tax for couples in the following way $\bar{T}_{1c} = \varepsilon$ and $\bar{T}_{2c} = 0$. The second option 2 modifies the tax for couples in the following way $\bar{T}_{1c} = 0$ and $\bar{T}_{2c} = \varepsilon$. The government faces a trade-off; under option 1 the outcome A eliminates the horizontal inequity, but B does not and under option 2 the outcome B eliminates the horizontal inequity, but A does not.

	Outcome A	Outcome B
Option 1	Horizontal equity	Horizontal inequity
Option 2	Horizontal inequity	Horizontal equity

With this example it is possible to see that the weight given to singles to avoid horizontal inequality will depend on the relative status of couples. In other words, the application of horizontal equity principle depends not only on the individual that will be affected with it but also on the status of those individuals that are indifferent.

The problem now is how to define the reference group. In other words how large the reference group to compare with should be, and where to place its boundaries. In order to solve this questions Auerbach and Hassett (1999) define an appropriate reference group for each income level. This reference group overlaps the individual's income, rather than assigning each individual to only one income class. The function proposed by Auerbach and Hassett is the following

$$W^{1-\rho} = \sum_i \left(\sum_k f_i (I_k - I_i) N_k \right) \tilde{I}_i^{1-\rho} (1-\tilde{t}_i)^{1-\rho}$$
$$\left[\frac{1}{\sum_k f_i (I_k - I_i) N_k} \sum_k f_i (I_k - I_i) \sum_j \left(\frac{I_k (1-t_{kj})}{\tilde{I}_i (1-\tilde{t}_i)} \right)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \tag{41}$$

Where $f_i(I_k\text{-}I_i)$ is the scaling function, which measures the difference between a household with income I_i and a household with income I_k is the reference group. And $\tilde{I}$ and $\tilde{t}$ are "representative" values for class i and $t_{kj} = T_{kj} / I_k$ is the average tax rate for household j of group i (for extension see Auerbach and Hassett; 1999).

Note that it is possible to think of a function where the measure of welfare is the utility function of the household instead of after tax income (individuals in a more general case). Where $u_i^j(c_i^j, h_i^j, I_i/w_t)$ is the utility of household j in the group i. Equation (36) can be written as

$$W = \left[\sum_{i=1}^{M} \sum_{j=s,c} \left[n_i^j (u_i^j (c_i^j, h_i^j, I_i/w_i)) \right]^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{42}$$

Equation (42) shows how society weights the utility function of the household within classes and across classes. This measurement of welfare can be adapted to a situation when one thinks that welfare should be measured in terms of utility levels. Letting $\bar{U}_k$ and $\bar{U}_i$ be utility levels in the absence of government intervention and U_k and U_i utility levels with government intervention the proposed measure of welfare is

$$W^{1-\rho} = \sum_i \left(\sum_k f_i(\bar{U}_k - \bar{U}_i) N_k \right) \tilde{U}_i^{1-\rho}$$

$$\left[\frac{1}{\sum_k f_i(\bar{U}_k - \bar{U}_i) N_k} \sum_k f_i(\bar{U}_k - \bar{U}_i) \sum_j \left(\frac{U_k)}{\tilde{U}_i} \right)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}}$$

IV. OPTIMAL TAXATION WITHOUT HORIZONTAL EQUITY CONSTRAINT

1. Optimal linear taxation of couples (Boskin and Sheshinski; 1983)

The Boskin and Sheshinski's (1983) model is an extension of the optimal linear income tax analysis of Sheshinski (1972) to analyze the optimal tax treatment of couple households. The main result states that the higher tax rate should be levied on the earnings with smaller elasticity of labor supply, which means that selective taxation is optimal since the elasticity of female labor supply is higher than that of male labor supply.

Households are assumed to have identical utility functions on total consumption, male's labor supply and female's labor supply $u(x_c^a, l_c^m, l_c^f)$. As in section 2.2, households differ in the male's productivity w_c^m and the female's productivity w_c^f. These productivity parameters have a joint distribution given by $f_c\left(w_c^m, w_c^f\right)$. Note that there is only one utility function for each household rather than one utility function for each member, as in section 2. In addition, a private good with economies of scale does not appear in the analysis. It means that my structure in section 2 is equivalent to Boskin and Sheshinski if $h_c = 0$ and the pure private consumption good is equal for both male and female. It means that $c_c^m = c_c^f$, which is aggregated in the household by one bundle of consumption given by $x_c^a = x(c_c^m, c_c^f)$.

The household faces the budget constraint from equation (10)

$$x_c^a = w_c^m l_c^m + w_c^f l_c^f - T_c(I_c^m, I_c^f) = T + (1-\tau_m) I_c^m + (1-\tau_f) I_c^f \tag{43}$$

where T is the lump-sum in a linear tax system and τ_i is the marginal tax rate on i's income $I_c^i = w_c^i l_c^i$. The problem of the government is to maximize a social welfare function W

$$W = \iint u(x_c^a, l_c^m, l_c^f) f_c(w_c^m, w_c^f) dw_c^m dw_c^f \tag{44}$$

subject to the resource constraint

$$T = \iint \left[\tau_m w_c^m l_c^m + \tau_f w_c^f l_c^f \right] f_c(w_c^m, w_c^f) dw_c^m dw_c^f - R \tag{45}$$

where R is the net required revenue. The government is also constrained by the fact that x_c^a, l_c^m, and l_c^f are functions of w_c^m, w_c^f and the tax parameters. For simplicity I follow Rees (2004), who assumes that the household utility function takes the quasi-linear form

$$v_c = x_c^a - u_c^m(l_c^m) - u_c^f(l_c^f) \quad \text{with } u_c'^i(l_c^i) > 0 \text{ and } u_c''^i(l_c^i) > 0 \ \ \forall i = m, f \tag{46}$$

which can be written in terms of incomes

$$u_c = x_c^a - v_c^m(I_c^m) - v_c^f(I_c^f) \quad \text{with } v_c'^i = u_c'^i / w_c^i \text{ and } v_c'^i = u_c''^i / (w_c^i)^2 \ \ \forall i = m, f \tag{47}$$

Note that the choice of utility function sets the effects of one partner's wage on the labor supply of the other to zero. Households maximize (46) subject to budget constraint (43). In this problem the demands are $x_c^a(T, \tau_m, \tau_f)$ and $I_c^i(\tau_i)$, and the indirect utility function is $v_c(T, \tau_m, \tau_f)$, with

$$\frac{\partial v_c}{\partial T} = 1; \ \frac{\partial v_c}{\partial \tau_i} = -I_c^i; \ \frac{\partial v_c}{\partial w_i} = (1-\tau_i) l_c^i \tag{48}$$

To find the optimal tax system I mix the individual problem's results with the government problem in equations (44) and (45). The first order condition with respect to the lump sum can be written as

$$\iint \frac{W'}{\lambda} f_c(w_c^m, w_c^f) dw_c^m dw_c^f = 1 \tag{49}$$

where $\lambda > 0$ is the marginal social cost of tax revenue and W'/λ the marginal social utility of income to a household with characteristic (w_c^m, w_c^f). Thus the optimal lump-sum tax equates the average marginal social utility of income to the marginal cost of the lump-sum (Sheshinski; 1972). The first order conditions on the marginal tax rates can be written as

$$\tau_i^* = \frac{\iint \left(\frac{W'}{\lambda} - 1 \right) I_c^i f_c(w_c^m, w_c^f) dw_c^m dw_c^f}{\iint I_c'^i(\tau_i^*) f_c(w_c^m, w_c^f) dw_c^m dw_c^f} = \frac{\operatorname{cov}(s, I_c^i)}{\overline{I}_c'^i} \tag{50}$$

where household marginal social utility of income W'/λ is denoted by *s*. There is no *a priori* reason to have $\tau_m^* = \tau_f^*$. Consequently, splitting income is very unlikely to be optimal. Empirical facts support that female's elasticity is higher than male's elasticity and consequently, with equation (50) a high marginal tax rate should be charged to males. However, the result of this model is not conclusive even if you accept the empirical facts. Though taxing women at a given rate creates a higher average deadweight loss than taxing men at the same rate, the policy maker's willingness to trade off efficiency for equity might imply that the tax rate on women could optimally be higher than that on men. This is because it may be possible that the covariance between the marginal social utility of household income and women's gross income is in absolute value sufficiently higher than that of men. In this sense the optimal tax analysis suggests a departure from income splitting, but it does not tell us much about the appropriate direction of this departure (Rees; 2004).

Boskin and Sheshinski's model is a particular case of the problem in section 2.3. First, this model considers only an optimal linear tax system. Second; it omits the private good with economies to scale *h*. In the same vein of Boskin and Sheshinski, Apps and Rees (1999) and Rees (2004) introduce the household production model in order to consider the intra-family decisions on consumption and labor supply.

2. Optimal linear taxation with household production model (Apps and Rees, 1999 and Rees; 2004)

As stated before, Rees (2004) extends the Boskin-Sheshinski's model to consider the household production model. Rees (2004) assumes that the female's wage is a monotonic function of the male's wage (Boskin and Sheshinski; 1983). Specifically, the female wage rate is an increasing function of the male, which takes the form

$$w_c^f = \delta w_c^m \quad \delta \in (0,1) \tag{51}$$

Now the male's productivity is written as $w_c^m = w$. In addition the household produces a domestic good *h* by using a production function $h_c = kg_c^f$. Where the productivity parameter *k* varies across households, and g_c^f is the time that female spends in domestic production. Consequently households face a joint density function $f_c(k,w)$. A value of the male's wage *w* corresponds now to a pair of wage rates. As before the problem is

$$W = \iint v_c(T, \tau_m, \tau_f) f_c(k,w) dkdw \tag{52}$$

subject to the revenue constraint

$$T = \iint \left[\tau_m \delta l_c^f + \tau_f l_c^m \right] w f_c(k,w) dkdw - R \tag{53}$$

The first order condition with respect to the lump sum T can be written as

$$\int \frac{W'}{\lambda} f_c(k,w)dkdw = 1 \tag{54}$$

Again denoting the marginal social utility of income to a household by T, the condition with respect to the i^{th} tax rate can be written as

$$\tau_i^* = \frac{\iint \left(\frac{W'}{\lambda} - 1\right) I_c^i f_c(k,w)dkdw}{\iint I_c'^i(\tau_i^*) f_c(k,w)dkdw} = \frac{\operatorname{cov}(s, I_c^i)}{\bar{I}_c'^i} \tag{55}$$

The result look similar to that derived in Boskin and Sheshinski's model. The denominator has the same meanings as before. The numerators present crucial differences with respect to the equation (55) above. The male's tax rate is unaffected by the introduction of household production, because I_c^m does not vary with k. However, the value of $\operatorname{cov}(s, I_c^f)$ now depends on the way in which female labor supply varies with k. Consequently, the government should implement a tax schedule for couples following the equation (55).

Three situations can be analyzed. If I_c^m is increasing with k the covariance is negative and it will be higher than that of males. It generates the possibility that the female optimal linear tax rate will be higher than the male's, essentially because it is a more powerful instrument for redistributing income from better off to worse off households. On the other hand, if I_c^m is decreasing with k, this covariance will be lower in absolute value than that for males, implying a negative tax rate on women. Finally in the case where both tax rates are positive, but female labor supply varies inversely with domestic productivity, there could be a great deal of vertical inequity in the tax system, essentially because the female income tax rate captures the effects of variation in domestic productivity only very imperfectly (Rees; 2004).

The results above are very interesting. However it is necessary to consider the analysis in a more general context. First, it is necessary to understand how the tax structure is under nonlinear tax function. Second, because in reality the government faces an economy with singles and couples, it is important to consider the mixed problem in the section 2.3.

V. THE AGENDA FOR FUTURE RESEARCH

The agenda for future research must try to solve a more general problem than those solved by the papers considered in this essay: particularly, the problem where singles and couples co-exist in the economy. The general problem is important for two reasons. First, it is not evident that the government can verify the single-couple status. Second, it presents a trade-off between horizontal and vertical equity because individuals differ in productivities and if they live as a couple or single household.

Three directions can be considered in future research. First, the problem requires a specific theory that explains how individuals within households take the intra-family decisions. The application of a specific theory implies a different informational structure, which may determine how individuals choose consumption, labor supply, level of public good and contributions for financing this public good. These choices imply the information structure in the problem and consequently, the incentive compatibility constraints. A good starting point is to analyze Schroyen's (2003) contribution. In this paper the incentive compatibility constraint depends on the trading possibilities within the household, in other words how the utility of each member can be affected by the possibility to trade with each member into the household.

The objective of this research topic must be to try to answer what type of tax function is the optimal one. In other words, whether households must face a joint, individual or selective tax system. Moreover, it is important to understand how this tax function should be. One of the issues of research is to know if the tax liability of the members within the household is different and who must face the higher one and consequently what would be its specific marginal tax rate.

The second topic of research must try to explain how the tax system can affect the incentive to live as a couple or to remain as a single. Apps and Rees (1999) and Schroyen (2003) do not explain how the tax policy parameters in the individual utility function may affect the incentives to create or destroy a household. In other words, if to live as a couple is considered as a coalition the literature does not explain how this coalition is created or destroyed. The starting point should consider the bargaining family theory. With this approach the problem should try to analyze how the tax system affects the participation constraint of each member to create the coalition.

Finally the issues of horizontal equity and vertical equity must be considered seriously: whether an individual should receive special treatment for the only reason that he lives in couple is not *a priori* clear. The analysis of Auberbach and Hasset (1999) and their measure of welfare adapted for the case when individual utility levels are the relevant concept on which inequality should be measured, is an important starting point on this issue.

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APPENDIX: HOUSEHOLD PRODUCTION THEORY UNDER LINEAR TAX

The household production theory introduces a domestic goods h_c^i, which is produced by each individual in the household $i = f, m$. Each good is consumed by both members of the household. This is a different approach to the one that introduces the intrafamily decision for good *h* in the problem; however it has been the more common methodology in the literature about taxation in couples. The household utility function is

$$u_c = x_c^a + \varphi(h_c^f) + \mu(h_c^m) \tag{A1}$$

where household has preferences on aggregate consumption x_c^a, domestic good h_c^f and domestic good h_c^m. I assume that $u_c(\bullet)$ is strictly concave, twice continuously differentiable, strictly increasing in x_c^a, h_c^f, and h_c^m. The household good h_c^f is produced according to the production function

$$h_c^f = kg_c^f \tag{A2}$$

where the productivity parameter *k* varies across households, and g_c^f is the female's time spends in domestic production.

Rees (2004) assumes that males in all households have the same productivity in household production. Since generally household production theory is interested in what is the effect of household female productivity on the labor supply. In this sense the domestic good h_c^m has a linear technology and depends only the time spent by males in its production, $h_c^m = g_c^m$. This theory defines a implicit price, *p*, to the domestic good h_c^f, which is equal to its marginal cost

$$p = \frac{(1-\tau_f)w_c^f}{k} \tag{A3}$$

Note that $(1-\tau_f)w_c^f$ is the after tax income of one unit of female time, which is used in the labor market. This cost is divided by the productivity parameter in the household production function equation (A2).

$$\frac{\partial p}{\partial \tau_f} = \frac{-w_c^f}{k} \tag{A4}$$

In the same way the price of h_c^m is $q = (1-\tau_m)w_c^m$. Individuals have the following time constraints

$$l_c^i + g_c^i = 1 \;\; \forall \;\; i = m, f \tag{A5}$$

where total time is normalized at 1. The household budget constraint is

$$x_c^a = w_c^m l_c^m + w_c^f l_c^f - T_c(I_c^m, I_c^f) = T + (1-\tau_m)I_c^m + (1-\tau_f)I_c^f \tag{A6}$$

which can be mixed with the time constraints, and getting

$$x_c^a + ph_c^f + qh_c^m = I, \;\; \text{with } I = T + (1-\tau_m)w_c^m + (1-\tau_f)w_c^f \tag{A7}$$

where the term *I* in the RHS is the total after tax income of the household. From this budget constraint it is clear that two households with identical male and female productivities and differing values of *k* will have different utility possibilities.

The problem of the households is to maximize (A1) subject to budget constraint (A7). In this problem the demands are $x_c^a(I,p,q)$, $h_c^f(I,p,q)$ and $h_c^m(I,p,q)$, and the indirect utility function is $v_c(I, p, q)$, with

$$\frac{\partial v_c}{\partial p} = -h_c^f; \;\; \frac{\partial v_c}{\partial q} = -h_c^f; \;\; \frac{\partial v_c}{\partial I} = 1$$

Households with higher value of *k* and equal female wage rate have lower value of *p* and consequently a higher utility level. What is most important in this analysis is the relation between female market labor supply and the productivity parameter *k* (see section 4.2 for an analysis with linear tax).