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Evaluating An Alternative To Finance Higher Education: Human Capital Contracts In Colombia

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Resumen. Se presenta un ejercicio para la valuación de Contratos de Capital Humano (CCH), siguiendo a Palacios (2004), en el cual se utilizan datos del Observatorio Laboral para la Educación y su Encuesta de Seguimiento a Graduados–2007. El análisis se hace a través de un modelo Minceriano y uno de Splines para encontrar los pronósticos determinísticos del ingreso. Se encuentra que los retornos a la educación superior proveen un incentivo para la implementación de CCHs para financiar completamente los programas de las universidades públicas y parcialmente en las universidades privadas. Financiar los programas de las universidades privadas requiere más ayudas para hacer los contratos rentables para los inversionistas y atractivos para los estudiantes.

Palabras clave: Retornos a la Educación Superior, Financiación Educación, Contratos de Capital Humano, Modelos de Splines Lineales, Regresión Intervalo.

Clasificación JEL: C01, I22, J24, J31.

Abstract. An exercise for Human Capital Contracts (HCCs) valuation is developed, following Palacios (2004), with estimations of future income created from data collected by the Labor Observatory for Education and its Following Graduates Survey-2007. The analysis is made through Mincerian and Splines models to derive income deterministic forecasts. The results show that returns to higher education provide an economic incentive for the implementation of HCCs to totally finance public university programs, and partially finance private university programs. Total financing of the latter still depends on additional aid to make the contracts both profitable for investors and attractive for students.

Keywords: Higher Education Returns, Education Financing, Human Capital Contracts, Splines Models, Interval Regression .

JEL classification: C01, I22, J24, J31.

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1. Introducción

Governments in developing countries face a restricted budget to compensate for underinvestment in education. Great advances have been made in the coverage of basic education, but this is not necessarily the case in higher education. In Colombia and other developing countries, individuals with academic potential might not have the financial means to access the higher education system, and governments have limited resources to address this potential demand. The loss of opportunity for each individual regarding his full potential is accompanied by the loss to society in productivity and welfare that is derived from an individual's investment in education. The present article seeks to reinforce the use of an instrument called Human Capital Contracts (HCC), linking private investors to those individuals as a proposed solution to the above-mentioned situation.

Recently, developments in the field of Finance have allowed the creation of different alternatives for long-term financing and investment, mainly through securitization and the deepening of world financial markets. These changes recall the idea of Friedman (1955) of investing in equity-like capital of individuals and their potential to generate income. Such investments take the form of Human Capital Contracts (HCCs), where individuals' future earnings are their collateral and source of resources to cover for an original investment.

Palacios (2004) makes a clear case for the importance of education in development, and introduces the historical process that HCCs have gone through, presenting them as a partial solution to the problem of access to education. HCCs allow resources from private investors to be transferred to students without financial means in exchange for a percentage of their future income. Palacios (op. cit) uses the results from a study on education returns by Núñez and Sánchez (2000) [1] to valuate a hypothetical HCC drawn up for implementation in Colombia.

This article drags heavily on Palacios (op. cit); however his analysis is taken one step further, focusing on the specific group of higher education graduates and their returns to education for HCC valuation. Previous studies in Colombia which focused on graduate students have tried to find determinants of graduate students' income (Forero and Ramírez 2008), but the aim of the article is to look exclusively at education returns for HCC valuation, and therefore to be the first study focusing on HCCs valuation in Colombia. Thereby, this article contributes to the literature of both Higher Education Financing and Economics of Education.

Here a simple model of HCC is introduced, in order to evaluate the feasibility of HCCs in Colombia from the perspective of the economic incentives for investors and students. According to the results, returns to higher education in Colombia are high enough to provide an economic incentive for the implementation of HCCs to wholly finance public university programs, and partially finance private university programs, given the information available.

[1] The cited example can be found in Palacios (op. Cit), who develops a model to valuate HCC to finance higher education in the Appendix A and C of his book.

This information is data from the Following Graduates Survey 2007 (FGS 2007) provided by the Labor Observatory of the Colombian Ministry of Education. A Modified Mincerian Approach and a Splines Model are used to estimate the returns to higher education, as they have been widely used in previous literature: Mincer (1974) for the US; Psacharoupoulos (1986) for a set of countries; Daniels and Rospabé (2005) for South Africa; Low et ál. (2004) for Singapore and finally for Colombia, Núñez and Sánchez (2000), Prada (2006) and García et Al. (2009).

I have applied some modifications to the Classic Mincerian Model. Following García et Al. (2009), and for most of the functional form, Heckman, Lochner and Todd (2008) criticism' to the Mincerian equation is incorporated: Working hours, gender, and linear splines are included in order to account for non linearities between wage and education. Here, age, instead of the potential experience is used in order to avoid the correlation between education and experience which affects the meaning of the education coefficient (Heckman et ál., 2008). Finally, we consider the life cycle earnings profile, built from the estimation, and compare the cash flow generated, to the direct costs of higher education in orther to valuate HCC.

This paper neither addresses topics on the distributional characteristics of graduates' income and the risk it involves when investing in HCCs. This might overlook risk aversion issues. Other changes to the classic functional form, like cubic, or quartic forms on experience (Weldi, 2006), or correcting for bias selection (Heckman, 1979) are not included either, for the sake of simplicity or due to the lack of proper information. Specifically, the correction of selection bias involves the inclusion of variables that might explain the labor market participation decision. Those variables include household information like number of household members, number of kids under certain age in the houshold, information that infortunatelly was not available in the FGS 2007. Not testing for selection bias might lead to biased estimators. Other problems related with the FGS sampling are related to the potential bias towards a certain group of institutions, as 25 % of the observations come from 4 universities, institutions which are thereby overrepresented in the sample. These issues are mentioned and acknowledged but not solved in the present article.

The article proceeds as follows: Section 2 presents a theoretical framework and reviews the most recent literature on education financing. Section 3 provides an overview of the Colombian higher education market. Section 4 presents the data from the FGS 2007. Section 5 presents the outcomes of the econometric exercises using OLS, Robust Standard Errors and Interval Regressions for the different model specifications. Section 6 presents Palacios (op. Cit) example to valuate HCC, and the estimation using the outcome of Section 5. Section 7 concludes.

2. Theorical Framework and Literature Review

2.1. Economics of Education

Education is viewed by some as an investment good (Mincer, 1958; Rebelo, 1991) in which individuals accumulate skills which are in turn matched by higher compensation in the labor market. That is the position of the Human Capital Theory (HCT). Education can also be seen as an investment that generates externalities, as it not only increases the productivity of the student but also the productivity of all factors (Lucas, 1988). Workers do not fully appropriate these because education has public good characteristics, according to the claim of the Endogenous Growth Theory (EGT). Lin (2007) analyzes the historical process of technological innovation that has evolved from exogenous shocks (that positively affected income) to endogenous ones produced by R&D made by people towards the technological frontier.

Education can also be seen as a consumer's good (Schultz, 1961): individuals who demand it are striving for status or recognition. Finally, there is a screening hypothesis (Spence 1973), whereby education actually signals the existence of pre-education workers' abilities. Thus, education is not productivity enhancing for all factors but can be seen as an efficient device to screen individuals' abilities and thereby their productivities..

Liquidity constraints affect the efficiency of market distribution of education. For the EGT and the HCT, solving the liquidity constraints on the students' side would help them to escape poverty traps as well as foster economic growth as a whole. Abiding by either, the screening or the education as consumption hypothesis, liquidity constraints would still act to prevent the attainment of optimal levels of education. In the case of Human Capital Contracts (HCCs), where private resources are transferred from savers to students, there is no debate regarding the positive effect on welfare enhancement that comes from their implementation. However, innovations of this nature might require some governmental support at the beginning, due to their risky nature. HCCs have the potential to correct these issues to a certain extent, so they are presented as a potential alternative to finance education.

2.2. Financing Education

Traditionally, societies have financed higher education with resources from the government-taxpayer and through direct financing of the individuals. By focusing on demand, government offers loans and subsidies to students or when focusing on supply issues, to Higher Education Institutions (HEIs). Direct financing can be made with savings and wages owned by individuals, their families and relatives; it also can be made through loans acquired in the private markets, which are often imperfect. Those resources are not enough to meet demand in developing countries.

As mentioned previously, governments in developing countries face tight budget conditions. Measures that increase supply without increasing the pu-

blic expenditure should be analyzed. There are two kinds of measures: those that increase efficiency of the resources already available, and those that increase the availability of resources. Competition mechanisms increase efficiency as universities are forced to compete for students, attracting them with improvements in their quality (i.e. vouchers). The availability of resources can be increased by graduate taxes and by involving the private sector in purchases of knowledge profitable in the market (research financing and student sponsoring).

For students, ideal measures would recover costs without damaging the access. As low–income students do not have the resources, they have higher levels of risk aversion –the returns of education are more uncertain for them and the opportunity cost to attend lectures is higher– and in most cases, they have no access to financial markets, which are actually imperfect for the case of education. If students know that they do not need financial resources to show up to classes and if they perceive that payments after graduation will not be unbearable, the measure will be successful. HCCs are a way for private markets to get more involved in filling the government's gap in financing higher education without marginalizing the low-income population (Palacios, op. Cit).

Currently, securitization allows the grouping of investors and students. Investors benefit from students being grouped, as the group income variance would be lower than the variance of each student taken separately. As predicted by the Portfolio Theory (Markowitz, 1952), investing in a pool of students will reduce the risk born by investors and will request a lower interest in return. Students also benefit from grouping because they pay less due to the effect of diversification and thanks to the fact that the risk of coercion by investors is reduced. Grouping students also spreads the administration costs over a wider population. As second markets grow and deepen, market information produces criteria to better judge performance of HEIs, as well as guide market demand for particular fields of study and any other derived from the grouping of students and investors.

2.3. The History of Human Capital Contracts

After considering the problems which arise when individuals intend to pursue higher education, and taking into account the irrevocable liquidity constraints, Friedman points out: "the device adopted to meet the corresponding problem for other risky investments (in other kind of capital) is equity investment plus limited liability on the part of the shareholder. The counterpart of education would be to "buy"a share in an individual's earnings prospects"(Friedman, 1955). Hereby the author established the main tenet of HCCs.

This tenet evolved into Income Contingent Loans (ICLs), first introduced experimentally by some universities in the USA. A known example was the Yale Tuition Postponement Program launched in 1970. Students' balances were grouped in such way that default students' balances would be added to the group balance; furthermore, maximum repayment period was 40 years. The program became burdensome for students, who perceived that it was too expensive and long termed. Although the experiment was not widespread, it drew

several criticisms concerning this kind of contract, and other aimed at the affordability of the maximum repayment period and the difficulty for HEIs to collect the loans (Palacios, op. cit).

The second wave of ICLs started in 1989 in Australia with a policy scheme called Higher-Education Contribution Scheme Program. Similar programs have been adopted by other countries (i.e Sweden, Ghana, Chile, New Zealand and the UK). In these programs, higher education is still highly subsidized and the tributary institutions make collection, in some cases. The ICLs idea veered from one Friedman's original one of investing in equity-like instruments to finance higher education.

By 1990, given the transformation of financial markets, with the explosion in the number of mutual funds and the increasing degrees of securitization of different assets, Chapman relaunched Friedman's idea. After lobbying for some legal frameworks, MyRichUncle^{TM} started operations in 2001 as the first institution investing in Friedman's idea. Past experience in the above-mentioned experiments has made a path for HCCs. Palacios (op. cit) introduces factors to consider in the legal contracts: specify the rights and duties of each party. He also includes special features such as exit conditions, caps for high-income students and provisions for low-income forgiveness or rearrangement conditions in case of eventualities.

2.4. Estimating Higher Education Returns

To valuate HCC, an estimation of the graduates' potential future income has to be made. Earnings equations, as proposed by Mincer (1974), have been widely used. In a such equation, the wage variable is explained with the scholar level measured in years of education attained, and with the potential experience and its quadratic form to account for the convexity of the relation.

Over the years, some modifications have been made to the original model. One of them tries to account for problems from the linearity of schooling (Splines Models). Linear spline models are used to replace the strong assumption that one additional year of primary school renders the same return as one of a doctoral program. Treatment of the experience variable has goes through modifications as well, given that working experience is hard to measure and thereby to be included in surveys. In several cases, the potential experience has been used as a proxy, defined as the difference between the age and the years of schooling. This specification makes the potential experience a function of the schooling years, distorting the interpretation of the schooling coefficient as the growth rate of income due to schooling. Such specification implies that log-earnings experiences are parallel across schooling levels, and that log-earnings age profiles diverge with age across schooling levels (García et Al., op. cit; Heckman et Al., op. cit). To avoid these problems, age is used as a proxy for experience. Furthermore, in order to break the assumption of unit elasticity of labor supply, the effective labor measure for the individual is included (measured as the log of monthly working hours).

In Colombia, some of the above mentioned specifications were recently tes-

ted by Núñez and Sánchez (2000), with OLS techniques for the period 1976-1998. They estimate a Mincerian equation and a modified version including linear splines for: primary school and finished primary; secondary and finished secondary; university and finished university and beyond.

Prada (2006), using OLS and Quantile Regression techniques, also estimates a Mincerian model separated from one with splines for data from 1985 to 2000. He finds that the returns to education present a cyclical behavior and that returns are heterogeneous depending on the quantile analyzed. Higher Education returns are found to oscillate between -2.5 % up to 46.8 %, depending on the economic cycle and the income group, with lower income groups correlated to higher variance.

By focusing only on higher education and using an earlier release of the database used in this article, Forero and Ramírez (2008) include all the variables they believe are determinants of income. Apart from the Mincerian variables they include employment characteristics, HEIs characteristics, and other socio demographical factors. Using OLS, Ordered Probit and Interval Regression estimates, the article finds that the following factors have a positive impact on the salaries of Colombian graduates: living in Bogota; being male; parental education; and holding a degree from private / accredited university. The fields of study and occupation areas also determine students' income.

For other countries, one of the most updated estimations is the one of Daniels and Rospabé (2005), which uses a Generalized Tobit Model for Interval Regression, which accounts for heteroskedasticity, in order to estimate a modified Mincerian earning function. Waldi (2006) uses panel data to establish differences among levels of education and gender; and also includes fields of study and other socio demographic criteria using a cubic form of the Mincerian equation. He studies the feasibility of HCC in Germany and finds that HCCs are able to finance partially undergraduate programs in this country. In order of comparability, it must be clear that in the present article, OLS, Robust Standard Errors and Interval Regression are used to estimate the returns to education in the group of the higher education graduates.

3. Higher Education in Colombia

According to Ayala (2006), the economic crisis at the end of the 1990s, severely affected the situation of higher education in Colombia. Although HEIs attendance rates for populations between 18-24 years increased from 12.25 % in 1993 to 16.5 % in 2003, the lag is noticeable when compared to the Latin American region as a whole. Only until 2003 Colombia reached the Gross Attendance Rates which Latin America as a whole had already reached in 1997 (25.7 %). The number remains far below the level of the OECD countries (54 % - 2003) and of the East Asian countries (Net at 24 % - 2000).

Currently, attrition rates reach levels of 60 %. Ayala (op. cit) suggests two main reasons: one, students get poorly prepared during High School; two, the acute crisis forced out students to the labor market. Another issue related to low attendance is the unused capability in private HEIs against overcrowding

in public ones. During the 1980s, 21 % of the capacity of private HEIs was not used; during the 1990s it got increased to 43 %, and up to 60 % in recent years. On the other hand, the ratio of acceptance to public HEIs has been between 10 % and 20 %, leaving out of the system 80 % of the applicants.

Private universities have increased their tuition fees more than proportionately, in order to cover their fixed costs with fewer students. Public universities receive resources from the central government, which are not adjusted for the increasing number of students: enrollment to public HEIs jumped 3.5 times since 1995, reaching 750,000 students in 2007. Furthermore, previous legislation has failed to come up with the promised resources: for example, regional entities have failed to make the transfers agreed to by law. However, all through the 1990s, public HEIs showed an increase in their own resources from 14 % to 20 %, mostly due to the lack of resources. By 2005, the average tuition fee cost was $359,000 (USD188) for new students of public universities. For the private ones, the tuition fees were in average $2'719,000 (USD1,426).

Icetex, a public institution, financed 11.46 % of the total tuitions in HEIs in 2003. But their resources are too scarce to fully meet the demand. In her article, Cárdenas (2003) presents other sources of funding available to students, which include consumer loans, credit cards and postdated checks, all of which are omitted here as feasible financial options, given that they are too expensive. In her article, she presents the credit lines for education from some institutions (Table 1). Banks offer conditions premised on long term definitions, which are too short for the requirements of higher education. The longest term is 5 years after period of study (6-12 months), and still 60 % of the credit has to be paid back while studying.

Table 1. Different Credit Lines to Finance Education

Entity	Credit Line	Financing Period	Interest rate	Credit Subject and other requirements
Bancafe – Davivienda	– Traditional	6, 12 months		
	– Credi U	60 % is financed during the study period (6,12 months) and the remaining balanced is defered up to 5 years	DTF + 13 % 23.14 %	Parentand Dependant – no codebto r required
Banco de Bogotá	– Short Term	6, 12 monmths	DTF + 8 %	Dependent, if he/s he has reached legal age, parents otherwise always – codebto r required
	– Long Term	50 % is financed during the study period (6,12 months) and the remaining balanced is defered up to 5 years	DTF + 12 % 22.2 %	
Banco de Crédito	– Crédito Universitario	6, 36 months	± 25 %	Student Parent guarantees the credit
Bancolombia	–Crediestudio	6, 36 months	27 %	Dependent, if he/s he has reached legal age, parents otherwise always – codebto r required

Continúa

Santander	–Universia	6, 12 y 6, 36 months for postgraduates studies	27 %	Parents as cosigners
Sudameris	– Educa T	6 months	25 %	Parents. Promissory notes or postdated checks required

(a.e.) Annual Efective

HCCs are an alternative to finance higher education in Colombia by using resources of the private sector. The expertise of the banking industry qualifies its members to take the initiative to offer HCCs, and to act as consultants to government-lead proposals.

4. Data Introduction and Description

This article uses data from the Labor Observatory for Education, their Following Graduates Survey 2007 (FGS 2007) follows the performance of graduate students' income. The FGS 2007 collects data from a sample who received their diplomas during the period 2000-2007. This data allows the evaluation of HCCs by providing additional specific features of the students and of the HEIs where they got their degrees.

Data includes 24,959 observations, from which 19,781 declare their income. The present section introduces data used in the econometric analysis. Although it is acknowledged that this data over-represents graduates from some universities, on this article sampling issues are not specifically addressed.

4.1. Income of the Graduates

Figure 1 shows the frequency distribution of graduates' income for the total sample and for different occupational positions.

Graduates' Income in the FGS 2007 is decomposed in seven monthly income intervals: [2]

1. Lower than $500 thousand (K) (USD263),
2. Between $500K and $1 million (Mln) (USD263-525),
3. Between $1Mln and $2Mln (USD525-1,050),
4. Between $2Mln and $3Mln (USD1,050-1,575),
5. Between $3Mln and $4Mln (USD1,575-2,100),
6. Between $4Mln and $5Mln (USD2,100-2,625)
7. Above $5Mln (USD3,150).

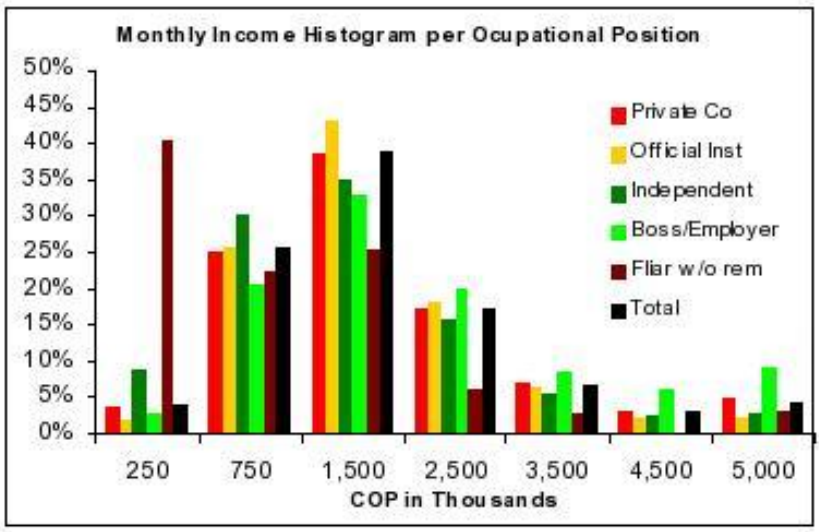
Excluding those observations related with individuals who declare to be family workers without remuneration, the analysis counts 19,714 observations. Colombian graduates are concentrated in the income interval between $1Mln and $2Mln (USD525-1,050) accounting for 39 % of the total, and as a whole,

[2]For USD conversion a rate of $1907/USD was used in the data introduction. Market Representative Rate (TRM) by August 29, 2008.

85 % of the sample have labor income that is lower than $3Mln (USD1,575). When classified by occupational position, employees of private companies happen to be placed more intensivly in high-income groups (over $3Mln) than public workers. The same pattern is present comparing self- employed workers with Bosses/Employers. However, in the highest income group this occupational position actually doubles the share of any other one.

Of the sample, male graduates account for 55 %. Going through the income intervals female gender' share happens to be decreasing with labor income: women in the lowest 3 intervals of income account for 64 %, 63 % and 58 % of the first, second and third income intervals, nonetheless, their shares on each one of the top intervals go down to 49 %, 43 %, 41 % and 29 %.

Figure 1. FGS 2007: Income Histogram



Source: FGS 2007-Labour Observatory for Education and Author's Calculations

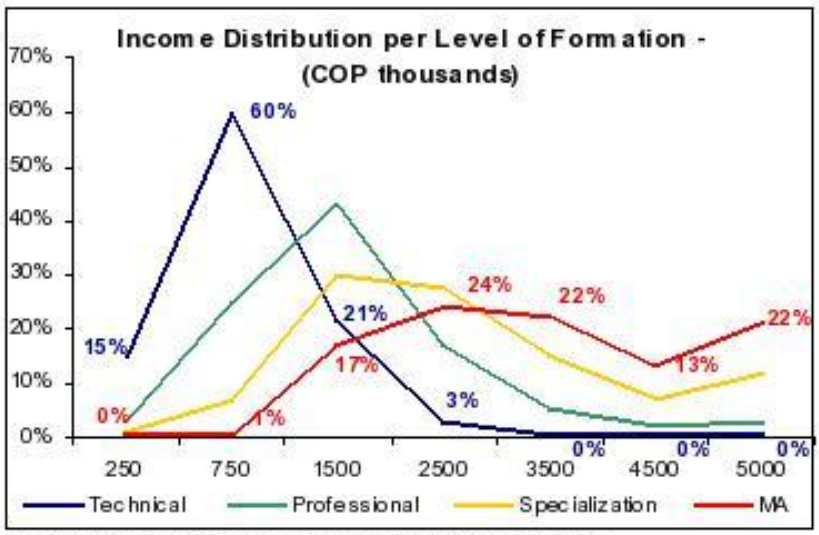
4.2. Education level

The structure of the graduate population per education level in the sub sample goes as follows: 75 % of the graduates finished their Bachelor degrees in formal universities, whereas 9 % obtained their diplomas from Technical and Technological (T&T) institutions. The remaining 15 % have pursued degrees higher than the professional level: Specialization (13 %), Master (MA-MSc) (2.4 %) and PhD (0.03 %). Following HCTs potential predictions and patterns, population with postgraduate studies is concentrated in higher income groups. Figure 2 shows the relative frequency lines for the different education levels. There are almost no observations for the T&T level in the higher income groups, as

there are no observations for the masters' level in the lower income groups. For PhDs, there are only 5 observations in the sub sample of the analysis. However, all the PhD observations in the sub sample are located in the intervals with incomes of $3Mln (USD1,575) per month or higher. Following the inverse relationship between income level, education level and gender which we already showed, education attainment is still marked by gender: the lower the education level, the higher the female concentration. For T&T and professional level, female observations account for 55 % and 56 % respectively; while in the MA level there are only 42 %, and from the 5 observations with PhD degrees, only one is a woman.

The differentiation among variables concerning initial and final income would help to prove the causality of education on income. In the present article, it is assumed that education causes the final income. The education level is provided in the Mincerian variable for years of schooling in one of the econometric models developed in the next section. Accordingly, the value is 14 years for T&T level, 16 for Professionals, 17 for Specializations, 18 for Master and 22 for PhDs.

Figure 2. Income and Education level



Source: FGS 2007-Labour Observatory for Education and Author's Calculations

4.3. Age Structure

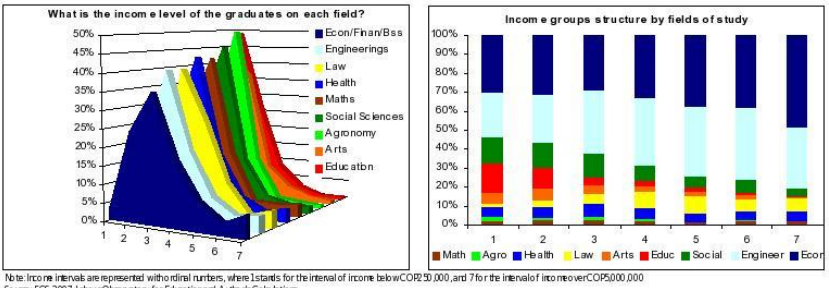
The mean of the graduates' age is 30 years, being centered in the group of 25-30 years (58 %); less than 10 % of the sub sample are younger than 25 years old; between 30 and 40 years of age, there are 26 % of the observations; and

over 40 only 7 %. For the T&T and the professional level the age average is 30 years old, while at the Specialization and Master levels the average is 35 years. As age is a proxy for experience, it should have a relationship to income and, in fact, the mean of age increases along the income intervals. At any given age the Master level has a higher income than any other education levels.

4.4. What do graduates study?

Another way to look at the distribution of income is to analyze its relation with the fields of studies. The FGS 2007 presents data for 55 different majors grouped into 9 fields: Agronomy, Veterinary and related; Arts; Education; Health Sciences; Law; other Social Sciences; Economics, Finance, Business and related; Architecture and Engineering; Mathematics, Natural Sciences and related. Graph 3 presents the income histograms for the different fields of study, whereas Graph 4 shows the structure of field of study within each income level. Both graphs present income intervals organized as ordinal numbers, where 1 stands for income under COP500K monthly; and 7, for income above COP5Mln.

Figures 3 and 4. Income and Field of Study: Histogram and Income Groups Structure Income and Education level



Note: Income intervals are represented with ordinal numbers, where 1 stands for the interval of income below COP250,000, and 7 for the interval of income over COP5,000,000
Source: FGS 2007-Labour Observatory for Education and Author's Calculations

It seems like there is an embedded decision about future income when students choose their field of study, which suggest a rational approach towards field of study and income. 69 % of the students chose to study Econ/Finance, Engineering or Law, fields showing higher means and a greater concentration in the higher income intervals. In the same way, less than 10 % of the population happens to chose either Math or Education, which have lower means and a stronger concentration in lower income intervals. The least demanded fields are Math and Agronomy, respectively 1,9 % and 1,3 % of the sub sample. This structure is similar to the data from the Ministry of Education: Agronomy, Math and Arts only have 8 % as the average of total subscribed in HEIs population during 2002-2007. Accordingly, the students from Economics, Engineering and Social Sciences including Law, accounted for 72 % of the total population, on average.

Studying Economics or Engineering does not indicate with certainty the

final outcome for income. Actually, these two areas represent more than half of the graduates regardless of the income group. However, the higher the income group, the higher income gets concentrated in these two fields: in the lowest income group they represent 54 % of the graduates, but they account for 81 % of the observations in the highest income interval. Law students are also more concentrated in the higher income groups: 1,9 % in the lowest income group and 6,1 % in the highest.

4.5. Features of Higher Education Institutions

According to the data, 25 % of the observations come from 4 HEIs, [3] an important observation to keep in mind before drawing any conclusions; data can be biased towards specific differences of these institutions as they are overrepresented in the sample. According to the Ministry of Education, [4] in 1995 enrolled students in Public HEIs accounted for 33 % (212,000 students) of all HEIs; by 2007 they represented 55 % (743,000 students). On average, during that period, 42 % of HEI students were registered at public institutions. The FGS (2007) does not capture this trend, and differs from the population distribution.

From the sub sample, 78,7 % of the individuals declare that they got their degrees private institutions. For each income interval and for each education level, private degrees become more relevant. The concentration is even more accentuated in higher income / education levels.

Colombian HEIs are mainly located in Bogota, according to the FGS 2007: they represent 37 % of the graduates of the sub sample. This is parcially due to the fact that they got their degrees here, as data from the Ministry of Education finds an average of 35 % of all graduates coming from Bogota's HEIs, for the period 2005-2007. In the FGS 2007 the lowest participation of graduates comes from both the Atlantic (Caribbean) region (6 %) and the Pacific region (1 %). Other regions have a more uniform distribution: Valle (16 %), Central (15 %), Antioquia (13 %) and the Oriental region (12 %). The Central region stands out by being the only region where higher education is mainly provided by the government: 73 % of the graduates in the region are from public HEIs. In Bogota, Antioquia, Valle and the Pacific, more than 90 % of the individuals surveyed graduated from private institutions. In the present article, Bogota is used as reference for both residence and for HEIs location.

4.6. Other variables

Working Hours, Place of Residence and Parental Education Attainment are all available in the FGS 2007 and they were included in the model as well. Regar-

[3] The universities are: Universidad de los Andes (1.905 Observations), Universidad de Caldas (1,851), Fundación Universidad de Bogotá Jorge Tadeo Lozano (1.539) and Universidad de San Buenaventura (1,038).

[4] Reference about the population subscribed in HEIs can be found in the Colombian Ministry of Education website at: http://snies.mineducacion.gov.co/men/index.htm

ding working hours, there seems to be a relation between the education level and the time spent at work: the higher the level, the lower the hours-worked mean value of the group. Considering Residence, although most graduates live in Bogota (36 %), the concentration of graduates variates with the level of income, with more concentration in Bogota for higher income intervals. The best income–performing graduates are living abroad: 28 % of the expatriates have an income higher than $5Mln (USD2,625) per month and 60 % of them are located in the three highest income intervals.

Finally, the FGS 2007 database presents 11 different levels of formation for parental education (Table 2). For the past generation, education attainment was higher for males and the relationship with the current generation is described as follows: graduates whose fathers attained a level of Academic High School or below represent 72 % of the T&T level graduates, 46 % of Professional level graduates; 49 % for specialized students; and at the MA level the percentage falls to 40 %. Parents' education level has an effect over the income of the descendants. Parents not only influence their offspring's decision about the final level of education, but also, after graduation, they help graduates with contacts and experience in the matching process.

Additional data available in the FGS 2007 is the information about the student's jobs. The data has variables for economic activity, the kind of contract they have with their employer, and some characteristics of the institutions they work for. Although these variables cannot be included deterministically in the valuation of HCC at certain values, since they are unknown at the time of the arrangement with a new student, still they give important insights on the final income and about the current demand for workers. For simplicity they were not included.

Table 2. Parental Education Attainment Levels

1. Primary School
2. Basic High School
3. Academic High School
4. Vocacional – Technical
5. Normal
6. Professional Technical
7. Technological
8. Professional
9. Specialization
10. Masters
11. PhD

Source: FGS – 2007
Dictionary of variables

5. Econometric Exercise

The present section aims to forecast income of higher education graduates, as the aim of the article as a whole is the valuation of HCCs. For that purpose some of the variables were transformed and some assumptions were made. For the working hours, the weekly variable is transformed into a monthly one by multiplying the mean of each interval by 30/7. Furthermore, the NA/NR observations for working time are located in the 31–48h interval. It is assumed that there is no loss of generality as this interval has the highest concentration of observations and as it does not change significantly the relations outlined in the previous section.

In their analysis, García et Al. (2009) include only data from wage earning graduates (Private Companies and Governmental Institutions), since they mutually compete in the labor market. The present article introduces an estimation and regression outcomes separately for this group. However, after clearing the problems that may arise with independent entrepreneurs who might declare less than earned income, HCCs would be offered to individuals regardless of their future occupational position. If we would include all the population in the analysis,it would increase the standard deviation of the estimates as the income of the population outside of the labor market has a more volatile outcome, and it also lowers the expected income as independent workers have a lower incomes according to the FGS. Both independent workers and employers are used here as a conservative measure, although the model does not fit well into this population (Appendix 4.1).

The present article, following García et al. (2009) for most of the functional form, incorporates Heckman et al. (2008) criticism to the Mincerian equation: Working hours, gender, and linear splines are included in order to account for non linearities between wage and education. Here, age is used instead of the potential experience, in order to avoid the correlation between education and experience, which affects the meaning of the education coefficient. Finally, we consider the life cycle earnings profile, built from the estimation, and compare the cash flow generated to the direct costs of higher education. So, a modified version of the Mincerian equation is presented along with a modified version of the previous Educative Splines models, both aiming to forecast graduates' income to valuate HCC.

The correction of the Sample Selection Bias suggested by Heckman et al. is not included, as there is not enough information on the household of the graduates to elaborate an estimation of labor market participation. Specifically, the correction of selection bias involves the inclusion of variables that might explain the labor market participation decision. Not testing for the significance of the error correction term might lead to biased estimators (Heckman, 1979). The parental education attainment variable is included to measure the correlation between previous generations' education attainment and the current generation level of income as a proxy for potential networks and to include the only available household information.

The methodologies used were OLS, Robust Standard Errors (RSE) and In-

terval Regression (IR) as it would be the case for a limited dependent variable. In cases where heteroskedasticity and non-normality of errors are present, thereby having consequences on the standard deviation of the OLS estimates and their t tests of significance, RSE is used. The present article follows the Consistent Covariance Transformation to estimate RSE. This procedure is suggested by the UCLA Statistical Consulting Group 2008, following White (1980).

On the other hand, IR accounts for the uncertain variance of the dependent variable when it is right, left or interval censored. In the FGS 2007 the dependent variable has this sort of characteristic, with right, left and interval censoring. For IR, the starting point model differs from the OLS:$y = X\beta + \varepsilon$. The model to estimate, when running an IR model is:

$$\vec{y} = X\vec{\beta} + \sigma\vec{\varepsilon} \tag{1}$$

where X is an $N \times k$ matrix including the independent variables, $\vec{y}$ is a vector representing the dependent variable responses, and $\vec{\varepsilon}$ is a vector of estimated errors with marginal survival distribution function $S(t)$, cumulative distribution function $F(t)$, and probability density function $f(t)$. That is, $S(t) = Pr(\varepsilon_i > t)$, $F(t) = Pr(\varepsilon_i \leq t)$, and $f(t) = \partial F(t)/\partial t$, where ε_i is a component of the error vector. The log likelihood, L, is written as below:

$$L = \sum \log\left(\frac{f(w_i)}{\sigma}\right), \quad \text{where} \quad w_i = \frac{1}{\sigma}\left(y_i - \vec{x}_i'\vec{\beta}\right) \tag{2}$$

If some of the responses are censored, the log likelihood can be written as:

$$L = \sum \frac{\log(f(w_i))}{\sigma} + \sum \log(S(w_i)) + \sum \log(F(w_i)) + \sum \log(F(w_i) - F(v_i)) \tag{3}$$

with the first sum going over the uncensored observations, the second sum over the right-censored observations, the third sum over the left-censored observations, the last sum over the interval–censored observations, and $v_i = \frac{1}{\sigma}\left(z_i - \vec{x}_i'\vec{\beta}\right)$, where z_i is the lower end of a censoring interval (Maddala, 1983). In the FGS 2007, there are no uncensored values and the first sum is not relevant. The estimations of the equation (3) parameters' are obtained through the Newton–Raphson algorithm. For all the models used on this article, the specifications converged.

5.1. Modified Mincer Estimation

In order to find the return for higher education as a whole and to look for splines among the higher education levels, we use two models to forecast the income of individuals. First, a Mincerian Model is analyzed, where the variable of schooling years, s, takes values of 14, 16, 17, 18 and 22, representing T&T, Professional, Specialization, Masters and Doctorate education, respecti-

vely. The above-mentioned variables are also included as follows:

$$\begin{aligned}\log(y_i) =& \alpha_0 + \rho_s s_i + \beta_1 Age_i + \beta_2 Age_i^2 + \beta_3 Gender_i + \beta_4 \ln(h\_month_i) \\ &+ \beta_5 Ed\_mthr_i + \beta_6 Ed\_fthr_i + \beta_7 HEI\_priv_i \\ &+ \sum \theta_a field_{a,i} + \sum \tau_b Rgn\_Rs_{b,i} + \sum \tau_c Rgn\_Grd_{c,i} + \varepsilon_i\end{aligned} \tag{4}$$

h_month_i: Individual Working Hours (Monthly)
Ed_mthr_i: Mother's education attainment
Ed_fthr_i: Father's education attainment
HEI_priv_i: Binary variable with value of 1 for private HEIs
$Field_{a,i}$: Binary variables for each field of study. Economics/Finance as reference.
$Rgn\_Rs_{b,i}$: Binary variables for each residence region. Bogota as reference
$Rgn\_Grd_{c,i}$: Binary variables for each graduation regions. Bogota as reference
$^*\sigma\varepsilon$, would be the error for the IR case.

Table 3 presents estimations for the coefficients. Significance of the included variables behaves similarly for the different methods. The direction and significance of the coefficients for the Classic Mincerian variables go as expected for all methods. Gender and the character of the HEI are determinant of the level of income, in favor of males and private institutions. Parental education attainment also influence the income of the graduates, but the education received by the father has a greater effect in the final outcome.

The elasticity of labor supply is low if compared with estimates found in previous articles (García et Al., op. Cit, find a 0,6 value for a population, that includes all kinds of education attainment). It should be noted that only graduates who declare income are being included; there might be a higher elasticity in the threshold at which individuals decide whether or not to enter into the labor force. A higher rate of growth for income is found with the IR method and so ti does for the coefficient of Age, but this rate also falls faster than the OLS estimate, as the coefficient of the square of age is lower.

The model shows that Engineers and Lawyers do enjoy a higher income (statistically significant) than Economists. The income of Health Sciences graduates is not significatively different from the Economists' one. Graduates from other fields earn less: the coefficients for graduates of Math and Agronomy fields are close to the reference; whereas this is lower for the graduates from other Social Sciences, Arts and Education. These results are similar to those found by Forero and Ramirez (op. cit).

Regarding geographic considerations, income is not significatively different in Antioquia with respect to the city of Bogota, but residence in any other region has a negative effect on income. The Colombians living abroad are an exception to this rule: their income is higher than graduates living in Bogota and the difference is high if compared with the negative coefficients of other regions. HEI's location is not as significant as the residence of the graduates. HEIs located in three of the seven regions defined (Valle, Central and Amazon) have incomes with no significant difference from graduates of Bogota's HEIs.

However, this may be misleading for the Amazon HEIs that only have 4 observations. The difference with Antioquia, the Pacific and the Atlantic regions is significant, in favor of Bogota's HEIs. Graduates from the Oriental region HEIs seem to have a higher income, but the coefficient of the IR is not significant.

Tests for heteroskedasticity, normality of the errors and multicollinearity of the independent variables were run. The White Test indicates that in the OLS estimations the hypothesis of homoskedasticity can be rejected. Also, according to the Anderson–Darling and the Kolmogorov–Smirnov tests, the hypothesis of normality of the errors can be rejected as well. In order to test for multicollinearity the variance inflation factor was checked for all the variables, and with the exception of Antioquia, Age and Age^2, all the VIF remained below 5; thus, multicollinearity was discarded.

The RSE were obtained as an attempt to correct the violation of homoskedasticity and normality assumptions. In most of the cases, the standard deviations are greater for each estimator but the significance remains unchanged. The overall model is significant and the F statistic is shown in Table 3 along with the R^2 for OLS and the Squared Multiple Correlation for the lower and upper bounds of the dependent variable for IR. The R^2 found here is in line with the findings in other studies.

5.2. Splines Model Estimation

A splines model is tested, on which the variable for years of education has been omitted. Instead, education splines are defined for three groups of the levels of formation: a binary variable with values 0 and 1 is used for the T&T level; another variable is included for post-graduate studies with values of 1, 2 and 6 for Specialization, MA and PhD, respectively, as those are years of schooling above the Professional level, the latter is used as the reference. Dichotomic vectors for level of study are used to evaluate interaction effects with other variables. Table 4 summarizes the definition of the variables included in the splines model. Splines' definitions follow what García et Al. (2009) did by analyzing all education levels; and in the present article, it is just for the higher education ones.

Following the above definitions, the Splines model is formulated below:

$$\begin{aligned}\log(y_i) =& a_0 + \rho_{\text{TC}}\, TEC_i + \rho_{\text{TC}} POS_i + \beta_1\, Age_i + \beta_2\, Age_i^2 + \beta_3\, Gender_i \\ &+ \beta_4\, \ln(h\_month_i) + \beta_5\, Ed\_mthr_i + \beta_6\, Ed\_fthr_i \\ &+ \beta_7\, HEI\_priv_i + \phi_1\, Age^2 * \delta_{TC} + \phi_2\, Age^2 * \delta_{PS,i} \\ &+ \phi_3\, Gender_i * \delta_{TC,i} + \phi_4\, Gender_i * \delta_{PS,i} \\ &+ \sum \tau_b\, Rgn\_Rs_{b,i} + \sum \tau_c\, Rgn\_Grd_{c,i} + \varepsilon_i\end{aligned} \tag{5}$$

$\delta_{TC/PS}$: d_tec and d_pos from Table 4
*$\sigma\varepsilon$, would be the error in the IR case.

Tabla 3. Modified Mincerian Model. Estimations

Parameter		OLS RSE IR	Parameter		OLS RSE IR	Parameter		OLS RSE IR	Parameter		OLS RSE IR
Intercept		7.0404 6.8128			-0.0750 -0.088			0.0301 0.0179			-0.0707 -0.0746
Std Errors	α_0	0.1176 0.1237 0.1221	Agronom		0.0354 0.0347 0.0368	Antioq		0.0257 0.0263 0.0264	HEI_Antq		0.0255 0.0267 0.0263
$t/Chi2$ value		59.8700 56.9125 3115.7			-2.1200 -2.1602 5.76			1.1700 1.1462 0.46			-2.7700 -2.6450 8.07
$Pr > \|t\| - Pr > ChiSq$		<.0001 <.0001 <.0001			0.0342 0.0308 0.0164			0.2405 0.2517 0.4967			0.0056 0.0082 0.0045
		0.2789 0.2881			-0.2354 -0.2674			-0.1870 -0.2057			0.0418 0.0418
yrs_edu	ρ_s	0.0053 0.0054 0.0055	Arts		0.0202 0.0211 0.0209	Valle		0.0225 0.0243 0.023	HEI_Vall		0.0220 0.0241 0.0225
		52.1800 51.7825 2696.97			-11.6600 -11.1614 163.99			-8.3300 -7.6973 80			1.9000 1.7330 3.46
		<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			0.0572 0.0831 0.0628
		0.0715 0.0764			-0.3865 -0.3912			-0.1611 -0.159			-0.0826 -0.0961
age		0.0052 0.0053 0.0053	Educatn		0.0186 0.0195 0.0193	Atlant		0.0276 0.0290 0.0282	HEI_Atla		0.0282 0.0291 0.0289
		13.7800 13.3769 204.35			-20.7900 -19.7787 408.82			-5.8500 -5.5518 31.78			-2.9300 -2.8372 11.06
		<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			0.0034 0.0046 0.0009
		-0.00075 -0.0008			0.0812 0.0709			-0.3305 -0.3466			-0.1606 -0.1313
age_Sq		0.0001 0.0001 0.0001	Law	θ_i	0.0176 0.0173 0.018	Pacific	τ_b	0.0446 0.0449 0.0459	HEI_Pacf	τ_c	0.0519 0.0528 0.0539
		-10.3900 -9.9987 113.46			4.6000 4.6887 15.48			-7.4100 -7.3549 56.95			-3.1000 -3.0435 5.95
		<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			0.0020 0.0023 0.0147
		0.1213 0.1394			0.0327 0.0295			-0.2405 -0.2515			-0.0042 -0.015
gender		0.0081 0.0081 0.0083	Healt		0.0173 0.0174 0.0178	Central		0.0213 0.0217 0.0219	HEI_Cent		0.0214 0.0215 0.022
		14.9900 14.9918 280.23			1.9000 1.8816 2.75			-11.3000 -11.0988 131.74			-0.2000 -0.1959 0.49
		<.0001 <.0001 <.0001			0.0578 0.0599 0.097			<.0001 <.0001 <.0001			0.8438 0.8447 0.4851
		0.2173 0.2111			-0.1622 -0.183			-0.1855 -0.2067			0.0409 0.0280
ln (h_month)	β_i	0.0079 0.0098 0.0082	SociSt		0.0133 0.0133 0.0138	Orient		0.0169 0.0165 0.0174	HEI_Orie		0.0167 0.0160 0.0171
		27.4200 22.1648 655.37			-12.1600 -12.1671 177.26			-10.9500 -11.2564 140.88			2.4500 2.5621 2.69
		<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			0.0141 0.0104 0.1013
		0.0133 0.0140			0.0564 0.0485			-0.1158 -0.1373			0.2065 0.1715
ed_mthr		0.0016 0.0016 0.0016	Engineer		0.0100 0.0099 0.0102	Amazon		0.0413 0.0401 0.0425	HEI_Amaz		0.2686 0.1750 0.2716
		8.3700 8.3208 73.33			5.6600 5.7022 22.37			-2.8000 -2.8857 10.43			0.7700 1.1797 0.4
		<.0001 <.0001 <.0001			<.0001 <.0001 <.0001			0.0051 0.0039 0.0012			0.4421 0.2381 0.5277
		0.0210 0.0222			-0.0931 -0.11			0.4795 0.5822			
ed_fthr		0.0015 0.0015 0.0015	Math		0.0293 0.0301 0.0302	Expat		0.0232 0.0249 0.0239	Root MSE		0.5322 0.5167*
		14.0900 14.0894 211.03			-3.1800 -3.0961 13.16			20.7100 19.2526 592.35			
		<.0001 <.0001 <.0001			0.0015 0.0020 0.0003			<.0001 <.0001 <.0001			
		0.0543 0.0559									
priv_HEI		0.0121 0.0119 0.0125									
		4.4900 4.5581 19.91									
		<.0001 <.0001 <.0001									

$F(OLS) = 355$ $Pr > F < ,0001$
Obs 19.714
Adj R Sqr 0.358359 0.3435*u
Adj R Sqr 0.3573 0.3154*l

Source: FGS – 2007. Author Calculations

* The pseudo R Squared used for Ir is the Squared Multiple correlation for the upper (u) and lower (l) of the income intervals.
** Scale σ of the IR Model.
In res, coefficients whit P values no significant at the 5 % confidence level.

It must be noted that there is no interaction between the binary variables for education level, $\delta_{\mathrm{TC/PS}}$, and the variable of the logarithm of the worked hours or the growth rate of income. Interaction with these variables was tested and the outcomes were counterintuitive and not significant. Interaction was also tested for the different fields of study and the effect does not seems to be significant for different levels of formation over the differences at the professional level. Table 5 presents the estimates for the Splines model.

Table 4. Education Splines. Definition of Variables

Educative Splain per level of education				**Complete degree premium**		
Form level	Sub level	TEC	POS	Level Form	d_tec	d_pos
Technical		14	0	Technical	1	0
University		0	0	University	0	0
Post–Graduate	Specialization	0	1	Specialization	0	1
Studies	Masters	0	2	Masters	0	2
	PhD	0	6	PhD	0	1

In this model, the value of the intercept also captures the expected return to education for a Bachelor Graduate. For comparison purposes $\alpha_0 + 16\rho_s$ from the Mincer model is provided:

- OLS and Robust Regression $= 11,5031$
- Interval Regression $= 11,4224$

These values do not seem too far from the intercept estimated through the splines model. Although the intercept is smaller in the IR regression, it seems to represent the same information as $\alpha_0 + 16\rho_s$ from the Mincerian estimation. Although both estimations are not comparable per se, the similarity in the estimations highlights the outcome.

Table 5. Spline Model Estimation

Parameter		OLS	RSE	IR	Parameter		OLS	RSE	IR
Intercept	α_{0+}	11.4041	11.2912				0.0000826	0.0001	
Std Errors		0.1015	0.1095	0.1048	dt_age2		0.0000304	0.00003	0
$t/Chi2$ value	ρ_{uni}	112.3700	104.1363	11603.5			2.72	2.485822	3.33
$Pr > \|t\| - Pr > ChiSq$		<.0001	<.0001	<.0001			0.0066	0.01293	0.0679
		-0.0436	-0.0405				0.0001269	0.0001	
TEC		0.0024	0.0025	0.0025	dp_age2		0.0000177	0.000017	0
	ρ	-18.4300	-17.5347	265.79			7.16	7.416521	51.88
		<.0001	<.0001	<.0001			<.0001	<.0001	<.0001
	Tec/	0.1613	0.1773				-0.0215	-0.0426	
POS	Pos	0.0176	0.0155	0.0181	dt_gen	ϕ_i	0.0268	0.0281	0.0281
		9.1400	10.4112	95.83			-0.800	-0.7645	2.29
		<.0001	<.0001	<.0001			0.4226	0.4446	0.1298
		0.0801	0.0863				0.0415	0.0693	
age		0.00534	0.0056	0.0055	dp_gen		0.0213	0.0202	0.0216
		14.99	14.2972	245.98			1.9500	2.0538	10.23
		<.0001	<.0001	<.0001			0.0511	0.040013	0.0014

Continúa

		-0.00092	-0.001				-0.0636	-0.0679
age_Sq		0.00008	0.00008	0.0001	Agronom		0.0355	0.0348	0.0369
		-12.21	-11.4772	160.95			-1.7900	-1.8276	3.4
		<.0001	<.0001	<.0001			0.0728	0.0676	0.0654
		0.1162	0.1307				-0.2328	-0.2599	
gender		0.0093	0.0094	0.0096	Arts		0.0202	0.0211	0.0209
		12.4900	12.3957	186.71			-11.5200	-11.0423	154.53
		<.0001	<.0001	<.0001			<.0001	<.0001	<.0001
		0.2164	0.2096				-0.3847	-0.3836	
ln(h_month)	β_i	0.0079	0.0098	0.0082	Educatn		0.0187	0.0195	0.0194
		27.3500	22.0668	648.81			-20.5900	-19.7074	390.25
		<.0001	<.0001	<.0001			<.0001	<.0001	<.0001
		0.0134	0.0142				0.0913	0.086	
ed_mthr		0.0016	0.0016	0.0016	Law		0.0177	0.0176	0.0181
		8.4200	8.3758	75.13			5.1500	5.1911	22.61
		<.0001	<.0001	<.0001		θ_i	<.0001	<.0001	<.0001
		0.0207	0.0223				0.0353	0.0387	
ed_fthr		0.0015	0.0015	0.0015	Health		0.0174	0.0175	0.0179
		13.9400	13.9707	212.05			2.0300	2.0162	4.68
		<.0001	<.0001	<.0001			0.0424	0.043794	0.0305
		0.056	0.0595				-0.1575	-0.1711	
priv_HEI		0.012	0.012	0.0126	SociSt		0.0136	0.0136	0.014
		4.580	4.665	22.44			-11.6100	-11.6119	149.53
		<.0001	<.0001	<.0001			<.0001	<.0001	<.000
$F(OLS) = 308{,}3$		$Pr > F$	< ,0001	Obs 19,714			0.0638	0.0612	
Adj R Sqr		0.3609	0.3476*u		Engineer		0.0101	0.0100	0.014
Adj R Sqr		0.3595	0.3159*l				6.3200	6.3744	34.85
Root MSE		0.5322	0.5155**				<.0001	<.0001	<.000
*Pseudo R Squared for IR is the Squared Multiple Correlation							-0.0813	-0.0903	
** Scale σ of the IR model					Math		0.0294	0.0300	0.0303
P values not significant at the 5 % confidence level in red							-2.7700	-2.7122	8.9
Source: FGS–2007. Author Calculations							0.0056	0.0067	0.0028

Parameter		OLS	RSE	IR	Parameter		OLS	RSE	IR
		-0.1867	-0.0262				0.0404	0.048	
Valle		0.0224	0.0242	0.023	HEI_Vall		0.0221	0.0243	0.0226
		-8.3200	-7.7040	80.68			1.8300	1.6627	4.51
		<.0001	<.0001	<.0001			0.0677	0.0964	0.0336
		-0.1558	-0.155				-0.0898	-0.096	
Atlant		0.0275	0.0289	0.0282	HEI_Atla		0.02832	0.029177	0.029
		-5.66	-5.38838	30.24			-3.17	-3.07729	10.97
		<.0001	<.0001	<.0001			0.0015	0.0021	0.0009
		-0.3325	-0.3538				-0.1611	-0.1287	
Pacific		0.0446	0.0450	0.0459	HEI_Pacf	τ_c	0.052	0.053	0.0538
		-7.46	-7.391	59.52			-3.110	-3.049	5.73
	τ_b	<.0001	<.0001	<.0001			0.0019	0.0023	0.0167
		-0.24016	-0.2527				-0.0057	-0.009	
Central		0.02126	0.0216	0.0219	HEI_Cent		0.0215	0.0216	0.0221
		-11.29	-11.1184	133.3			-0.2700	-0.2640	0.16
		<.0001	<.0001	<.0001			0.7905	0.7918	0.6854
		-0.1864	-0.2074				0.0410	0.0353	
Orient		0.0169	0.0164	0.0174	HEI_Orie		0.0168	0.0162	0.0173
		-11.02	-11.3371	142.33			2.4400	2.5347	4.17
		<.0001	<.0001	<.0001			0.0148	0.0113	0.0411
		0.4844	0.5886						
Expat		0.02313	0.024782	0.0239					
		20.94	19.54516	607.22					
		<.0001	<.0001	<.0001					

P values not significant at the 5 %
confidence level in red
Source: FGS–2007. Author Calculations

In the Spline model, all the socio economic variables are strongly significant as well, and the values of the estimators are close to the Mincer model ones. Both gender and the character of the institution have an effect over income in

favor of males and private institutions. Parental education level also positively affects the expected income of the graduates, and again, the father's one has a higher impact. As expected, the logarithm of monthly worked hours also impacts income positively. The place of residence has a similar effect as in the Mincerian model, with the department of Antioquia having no significant difference with Bogota and all other regions significant with a negative effect, the only exception being the graduates living abroad: their coefficient is strongly positive. The main difference with the Mincerian model dwells in the speed income growths measured with the coefficients for Age and Age^2: by using the professional level as reference, the income is expected to increase more rapidly but the rate at which it increases is expected to stabilize faster than in the Mincerian model.

As expected, having only finished the T&T level has a negative effect over income when compared whit having a Bachelor degree: finishing a post-graduate level has a positive effect. They are both strongly significant. Given the coefficients of the interaction variables, it seems that the speed at which the rate of income decreases is slower at the T&T level, and for the post–graduate levels. However, in the T&T level the degree of significance is not strong and even insignificant in the case of the IR estimation. When gender gaps at each education level are tested, findings show that at the T&T levels the interaction coefficient is not significant. At the level of postgraduate studies, the gap in favor of males seems to be deeper than at the professional level; the coefficients are significant but not strong, and they are non–significant with OLS.

In the OLS estimation, all the variables' VIF remain under 5 but for Age and Age^2, so multicollinearity is discarded. The assumptions of normality of the errors and of homoskedasticity are rejected again, according to the respective tests; so, RSE are relevant for the splines model as well. The models are overally significant according to the F statistic and its outcome is presented in Table 5 along with the R Squared and the Squared Multiple Correlation for the IR.

5.3. Robustness Exercises

A Forward Selection model was used to test the relevance of the included variables. This technique starts with no variables and adds them one by one, according to the contribution to the model measured by the increase in R^2. In the case of the Modified Mincerian model, the Forward test leaves aside the variable representing HEI located in the Central Region. The variable is kept within the Model as it is part of a greater categorical variable.

For the Splines Model, the interaction effect of Age with the dichotomic variables for levels of formation proves to be statistically insignificant and it is not kept. In the previous sub section the coefficients are already removed and not presented in Eq.(5.5). All other variables are kept and the Forward Selection model is run again, with 35 steps out of 37 variables: the HEI Central and HEI Amazon variables are left aside.

Both models were run only for wage earning observations and for the Independent and the Boss/Employer observations. Those who declare to have an

occupational position inside of a Private Company or in Public Institutions compete in the labor market and are subject to its rules (García et al., op. cit). Furthermore, income from individuals who are not in the labor market might be determined by other variables different than education, and the Mincerian or Splines Model should not adjust as well to the data.

Estimates of the wage earning observations have higher intercepts and higher growth rate for income (Age coefficient) in both the Mincerian and the Splines models. The data seems to explain the wages better, as the R^2 is higher for both models in all methods. In the next section they are not used, first, due to a conservative measure, given that coefficients for the intercept and income growth are higher; and second, due to the fact that the information about the future occupational position of a student is not available when HCCs being arranged. The estimations for both, wage earning and non–wage earning observations are presented in Appendix A.1.

Forecasts of income at graduation were estimated to be used in the proceeding section with their respective confidence intervals following the outcome of the regressions introduced in this section. A summary of calculations is presented in Appendix A.2.

6. Valuating Human Capital Contracts in Colombia

If the goal of HCCs is to link private investors with students, it can be argued that a demonstration of their profitability has to be made before any discussion is carried out about the legal, institutional arrangements, and implementation issues of HCCs. Nevertheless, even without this demonstration, government can use the analysis to address the issue of retributive taxes for public universities' graduates. Any tax has to be inside the range that HCCs determine. It is expected that the focus on higher education graduates increases the feasibility of HCCs, as opposed to studies focused on the return to education for the whole population.

Right now, we will make a brief presentation of Palacios' (op. Cit) HCCs valuation model, followed by a brief review of the assumptions used. Then, a Mincerian transformation will be used to estimate the potential viability of HCCs implementation in Colombia, using the forecasts obtained in the previous section.

6.1. Pricing Human Capital Contracts (HCCs)1

The value of one HCC is mainly determined by the expected value of the income that the student(s) will generate during the agreed time of the contract, and the percentage of income investors will derive from the operation. Eq.(6) establishes the relationship: γ stands for the percentage of the present value of income (PVI) to which the students commit when they sign the contract; $(1-u)$ is the probability of employent of that student,u being the unemployment rate for the specific groups of higher education graduates; a is a parameter representing administrative costs generated by issuance and collection; and d represents the

expected cost of default:

$$HCCV = \gamma \cdot PVI\,(1 - (u + a + d)) \tag{6}$$

Students will start repaying their obligations after s years of schooling and the contract will have a repayment period of k years. Using continuous compounding and an interest rate i to discount the cash flows, the PVI can be defined as $PVI = \int_s^{s+k} Y(t)e^{-it}dt$. and $G(t)$ is the income growth function:

$$PVI = Y_s e^{-is} f\,(i, k, G(t))\ , \tag{7}$$

$$\text{where} \quad f\,(i, k, G(t)) = \int_0^K G(t) \cdot e^{-it} dt\ .$$

Substituting (7) in (6), the value of HCC includes the term for the expected income upon graduation.

$$HCCV = \gamma \cdot Y_s \cdot e^{-is} \cdot f\,(i, k, G(t))\,(1 - (u + a + d))\ . \tag{8}$$

The profit that an investor receives from a HCC is given by $\pi = HCCV - C$, where C is the amount financed by the investor. However, in a competitive environment, the profits of the contract would be zero and the risk premium would be included in the competitive interest rate used to price the investments of similar risk. Thus $HCCV = C$ and the percentage of income to be committed by the students would be:

$$\gamma = \frac{C \cdot e^{is}}{Y_s\, f\,(i, k, G(t))\,(1 - (u + a + d))}\ . \tag{9}$$

Equation (9) shows that if income at graduation or the potential for income growth are high, then the percentage of income that should be committed will be lower; similarly, higher costs derived from the operation or harder conditions for graduates to get employed will both create a greater risk which should be compensated by a higher percentage of income to be committed in the HCC. The Mincerian Solution suggested by Palacios (op. Cit) is presented in Appendix A.3.

6.2. Macroeconomic Assumptions over the Parameters

A summary of the assumptions made by Palacios (op. Cit) is presented in Table 6, and the update used in this article is presented in Table 7. For comparison purposes this article will also set the example in USD. The 2005–2007 average exchange rate is used as an update. The interest rate to be charged in these sorts of investments, suggested by Psacharoupoulos (1986) is 8 %. 10-Years Colombian Government spread over the US Treasuries has decreased to 425 bps, [5] making investments in Colombian assets less risky; the assumption of

[5] Information from http://www.bloomberg.com/markets/rates. Consulted on September 1st, 2008.

3 % inflation in the US is kept. However, it is worth mentioning that Colombian mortgages use a Real Value Unit (UVR in Spanish acronym) to price long-term contracts as mortgages and some government inflation indexed bonds. When implementation is discussed, the use of UVR can clear students and investors from risks related to the both exchange rate and the inflation.

Table 6. Assumptions over the Parameters (Palacios, op. cit)

Parameter			Source
FX Rate	1427.04	Average daily 1998	Banrep (1)
Unemployment	9.10 %	Average u 1991–2000	Dane (2)
Default	15 %	College graduates in the informal sector + 5 %	Nuñez (2000) (3)
Discount rate	8 %	Based in Psacharoupoulos	Psacharoupoulos (1986)
Premium	5 %	Country risk	S&P and Citigroup (4)
Nominal	13 %		
Inflation	3 %	Inflation Estimates	Palacios (2004)
Risk adjusted real interest rate	10 %	Continuous compounding	
Repayment period	10	Years	Assumption
Time to start repayment	5	Career Years (Universitary)	
Administratn Cost	2.0 %		Assumption
USD financed yearly	2382.55		Lopez (2001)

(1) Banrep stands for Banco de la República de Colombia. Central Bank.

(2) This is a conservative measure as the graduates from higher education levels have lower unemployment rates.

(3) Nuñez (2000) find tha 90 % of the graduates from higher education levels declare income or have Social Security.

(4) Nov 01. Colombian Spread for 10Y Bonds over the US Treasures was 504 bps.

Unemployment and default rates include some conservative additions in both Palacios' example and the exercise made here. For his example, the unemployment rate is the one provided by the National Statistics Department (DANE), for the population with age between 25-55 years old, as most of the repayment period is expected to fall within this interval. Unemployment for graduates of higher levels of education is lower; here the average 2001-2007 data for national unemployment will be used from the Household Survey (GEIH) provided by DANE. Palacios (op. Cit) sets the default rate based on Núñez (2000), who estimated that 90 % of the graduates from HEIs are in the formal sector, either declaring income or making contributions to the Social Security System. Those income-tracking records can assess the reliability of information provided. As some graduates in the informal sector would pay and some in the formal sector would default, the Default rate is set at 15 % as a conservative measure. In the present article the default rate has been set at the same level.

Table 7. Present Assumptions over the Parameters

Parameter			Source
FX Rate	2252.55	Average monthly 2005–2007	Banrep (1)
Unemployment	13 %	Average u 2001–2007	Dane (2)
Default	15 %	College graduates in the informal sector + 5 %	Nuñez (2000) (3)
Discount rate	8 %	Based in Psacharoupoulos	Psacharoupoulos (1986)
Premium	4.25 %	Country risk	Bloomberg (4)
Nominal	12.25 %		
Inflation	3 %	Inflation Estimates	Palacios (2004)
Risk adjusted real interest rate	9.25 %	Continuous compounding	
Repayment period	10/15	Years	Assumption
Time to start repayment	5	Career Years (Universitary)	
Administratn Cost	2.0 %		Assumption
USD financed yearly	2400.00	Average tuition per semester in private HEIs: COP2.7mll	Ayala (2006)

(1) Banco de la República de Colombia. Central Bank.

(2) Household Survey. Consulted Aug 2008.

(3) Nuñez (2000) find tha 90 % of the graduates from higher education levels declare income or have Social Security.

(4) Sep 08. Colombian Spread for 10Y Bonds over the US Treasures was 425 bps.

An update of the tuition costs from Ayala (2006) is used here: the average tuition per semester in private HEIs was $2.7Mlns (USD1,199). Discussion over the repayment period should take into account: first, a private incentive to allocate resources into students' education is needed; second, that the percentage committed to by students should not be prohibitive for them (MyRichUncle^{TM} has a limit of 15 %); and third, that long repayment periods have already proved unsuccessful. Here, repayment periods of 10 to 20 years were considered.

With Palacios' set of parameters and the estimates of returns to education from Núñez and Sánchez (2000), an outcome measured by Eq.(9) and Eq.(A.8) can be derived introducing the Mincerian solution suggested by Palacios (op. Cit) and presented in Appendix 3. A summary of calculations is presented in Table 8. If a student wants to finance his tuition for his last year of education, he would be required to commit 9,11 % of his future income; alternatively, if he wants to finance his first year, having to wait longer to start repayment, he would have to commit 13,6 % for a contract within a 10 year repayment period. Under the assumption that other conditions do not change, a male student who wants to finance his whole career would have to commit 56 % of his future income.

Table 8. (γ) % of Income to commit in HCC. Palacios (op. cit) using N&S 2000 estimations

$\rho = 0{,}102$	$C = USD2382{,}56$		Mincerian	
Time left to repayment (5–s)	Men		Women	
	$k = 10$	$k = 15$	$k = 10$	$k = 15$
1	9.11 %	7.11 %	13.88 %	10.83 %
2	10.07 %	7.85 %	15.35 %	11.97 %
3	11.13 %	8.68 %	16.96 %	13.22 %
4	12.30 %	9.59 %	18.74 %	14.61 %
5	13.60 %	10.60 %	20.71 %	16.15 %
Total Cumulative	56.21 %	43.83 %	85.65 %	66.78 %

Source Palacios (2004), Núñez and Sánchez (2000) and Author's Calculations

6.3. HCCs with data from the FGS 2007

Using the estimates from the econometric models applied in the present article and with the parameters assumed in Table 8, HCCs are still a tool to finance, at partially least, higher education, if the maximum percentage of income that a student is able to commit to finance education goes around 15 % to 20 %. Table 9 presents the main estimations from HCCs valuated for different sets of income at graduation, with the outcome of the previous Modified Mincerian Model (IR). Outcomes using the estimations from OLS and the Splines model can be found in Appendix A.2. Depending on the length of the contract and the field of study under analysis, considering the available information, one HCC is capable of financing 3 up to 4 years of university studies. The field of Law ranks highest in the valuation of HCCs.

The findings suggest that the estimation of future income of a university graduate requires further research. On average, the income of university graduates is not enough to cover the risk embedded in HCCs and to attract investors without making the contract prohibitive for students. However, including variables that reflect the ability of a student (i.e. high school grades, subjective appreciations), and that are able to determine future income after graduation, may lead to the possibility of a more favorable valuations for students.

The focus on the group of graduates from HEIs is the factor that allows measurement of the way labor income increases in relation to the educational attainment. However, the increase in the cost of higher education at the private sector makes the investment very risky as the cost is known with certainty, but the return is not. On the other hand, tuition in public institutions is 7,5 times lower than in private ones. When the capability of HCC is tested to finance public education, the outcome is completely the opposite, and financing the tuitions with private resources should be attractive for investors given the expected return. Table 10 presents the outcome for HCCs with only 10 and 15 years of repayment period. Less than 5 % of the income of the student is required for any of the presented fields. As education in public institutions is

cheaper and as the return to education is high enough, more investors might be attracted to support students with very low income and high opportunity costs, students who otherwise would have to be looking for a job, most likely within the informal sector.

Table 9. HCC for Private Universities, (γ) % of Income to be committed by Students. (From Mincerian Model – Interval Regression)

Time left to repayment (5–s)	10 Years Repayment Period							
	ECONOMICS		HEALTH SCIENCES		ENGINEERING		LAW	
	Male	Female	Male	Female	Male	Female	Male	Female
1	5.3 %	6.0 %	5.0 %	5.8 %	5.0 %	5.7 %	4.8 %	5.5 %
2	5.8 %	6.6 %	5.5 %	6.3 %	5.4 %	6.3 %	5.2 %	6.0 %
3	6.3 %	7.3 %	6.0 %	6.9 %	6.0 %	6.9 %	5.7 %	6.6 %
4	6.9 %	8.0 %	6.6 %	7.6 %	6.5 %	7.5 %	6.3 %	7.3 %
5	7.6 %	8.8 %	7.2 %	8.3 %	7.2 %	8.3 %	6.9 %	8.0 %
Cumulated total	31.9 %	36.7 %	30.3 %	34.9 %	30.1 %	34.7 %	29.0 %	33.4 %

Time left to repayment (5–s)	15 Year Repayment Period							
	ECONOMICS		HEALTH SCIENCES		ENGINEERING		LAW	
	Male	Female	Male	Female	Male	Female	Male	Female
1	4.0 %	4.6 %	3.8 %	4.4 %	3.8 %	4.4 %	3.6 %	4.2 %
2	4.4 %	5.1 %	4.2 %	4.8 %	4.2 %	4.8 %	4.0 %	4.6 %
3	4.8 %	5.6 %	4.6 %	5.3 %	4.6 %	5.3 %	4.4 %	5.1 %
4	5.3 %	6.1 %	5.0 %	5.8 %	5.0 %	5.8 %	4.8 %	5.5 %
5	5.8 %	6.7 %	5.5 %	6.4 %	5.5 %	6.3 %	5.3 %	6.1 %
Cumulated total	24.4 %	28.1 %	23.2 %	26.7 %	23.0 %	26.5 %	22.1 %	25.5 %

Time left to repayment (5–s)	20 Years Repayment Period							
	ECONOMICS		HEALTH SCIENCES		ENGINEERING		LAW	
	Male	Female	Male	Female	Male	Female	Male	Female
1	3.5 %	4.0 %	3.3 %	3.8 %	3.3 %	3.8 %	3.1 %	3.6 %
2	3.8 %	4.4 %	3.6 %	4.2 %	3.6 %	4.1 %	3.4 %	4.0 %
3	4.2 %	4.8 %	4.0 %	4.6 %	3.9 %	4.5 %	3.8 %	4.4 %
4	4.6 %	5.3 %	4.3 %	5.0 %	4.3 %	5.0 %	4.1 %	4.8 %
5	5.0 %	5.8 %	4.8 %	5.5 %	4.7 %	5.4 %	4.5 %	5.2 %
Cumulated total	21.0 %	24.1 %	20.0 %	23.0 %	19.8 %	22.8 %	19.0 %	21.9 %

Source: FGS–2007 and Author's Calculations

Table 10. HCC for Public Universities, (γ) % of Income to be Committed by Students. (From Mincerian Model – Interval Regression)

Time left to repayment (5–s)	10 Years Repayment Period							
	ECONOMICS		HEALTH SCIENCES		ENGINEERING		LAW	
	Male	Female	Male	Female	Male	Female	Male	Female
1	0.8 %	0.9 %	0.7 %	0.8 %	0.7 %	0.8 %	0.7 %	0.8 %
2	0.8 %	1.0 %	0.8 %	0.9 %	0.8 %	0.9 %	0.8 %	0.9 %
3	0.9 %	1.0 %	0.9 %	1.0 %	0.9 %	1.0 %	0.9 %	1.0 %
4	1.0 %	1.1 %	1.0 %	1.1 %	1.0 %	1.1 %	0.9 %	1.1 %
5	1.1 %	1.3 %	1.1 %	1.2 %	1.1 %	1.2 %	1.1 %	1.2 %
Cumulated total	4.7 %	5.3 %	4.5 %	5.1 %	4.4 %	5.0 %	4.3 %	4.9 %

Time left to repayment (5–s)	15 Years Repayment Period							
	ECONOMICS		HEALTH SCIENCES		ENGINEERING		LAW	
	Male	Female	Male	Female	Male	Female	Male	Female
1	0.6 %	0.7 %	0.6 %	0.6 %	0.6 %	0.6 %	0.5 %	0.6 %
2	0.7 %	0.7 %	0.6 %	0.7 %	0.6 %	0.7 %	0.6 %	0.7 %
3	0.7 %	0.8 %	0.7 %	0.8 %	0.7 %	0.8 %	0.7 %	0.7 %
4	0.8 %	0.9 %	0.8 %	0.9 %	0.7 %	0.8 %	0.7 %	0.8 %
5	0.9 %	1.0 %	0.8 %	0.9 %	0.8 %	0.9 %	0.8 %	0.9 %
Cumulated total	3.6 %	4.1 %	3.5 %	3.9 %	3.4 %	3.8 %	3.3 %	3.8 %

Source: FGS–2007 and Author's Calculations

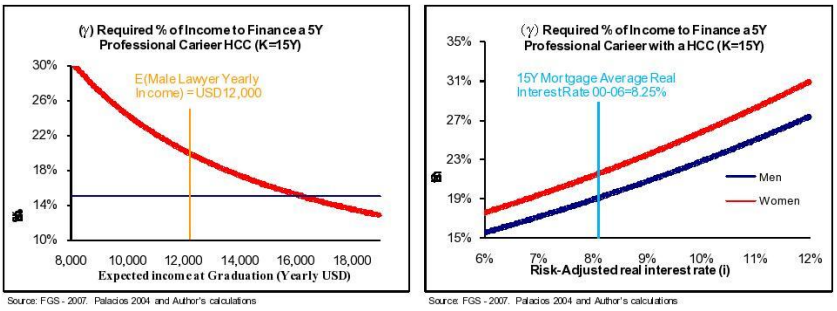
The outcome presented here does not reject the possibility of financial support for students willing to enter a private university using HCCs. Instead, HCCs can still be competence-generating and with support of the government, they can be brought into practice. For example, this can be done by allowing students to commit to a maximum amount of their income (i.e. 15the government to subsidize the remaining balance, through Icetex or another institution of its kind. Such an implementation would ease the pressure on those private universities that had to raise their tuition during recent years; while at the same time would allocate the resources spent by the government in higher education more efficiently.

6.4. Further Comments about HCCs in Colombia

According to the current information, HCCs are able to partially finance investments in education in private institutions and totally in public institutions. However, their use can lead to an increase in the amount of rejected students in comparison with the aspirants; thus, it would not help in the distribution of private institutions' unused capacity and would add pressure on the demand for spots in public institutions, whose supply follows the government's criteria. It has already been mentioned that HCCs use might help set a retributive tax for graduates from public institutions; also, combined with government subsidies, HCCs can enhance competition and redirect resources to private institutions, whose capacity is not fully used.

Plenty of students would be able to be financed by HCCs, but the variables used here are not able to identify which ones. Requesting more information from graduates in next releases of the FGS can move towards the direction of obtaining more specific information. On the other hand, the present article implemented several conservative measures which, jointly, might be affecting HCCs valuation. Thus, HCCs sensitivity has been checked through the comparative statistics on the variables that affect their value.

Figure 5.
(γ) – E(Income at graduation) (γ) – Risk Adjusted Real Interest Rate



The main driver of HCC value is expected income at graduation. Figure 5 presents the relation between (γ) and the expected income at graduation, using OLS Mincerian estimation for h and g (income growth rate and income decreasing speed rate) and keeping all other parameters constant. Students who earn USD15,500/year can theoretically commit to HCCs with an acceptable 15 % or less of their future earnings. For comparison purposes, it is worth mentioning that male lawyers in Colombia on average have a yearly income of USD12.000 at graduation (Appendix A.2).

Figure 5 also illustrates the relationship between the interest rate and the percentage of income to be committed to in HCCs. Measures which reduce the real interest rate make HCCs affordable to students. Either valued in USD or UVR, control of devaluation/inflation and country risk profile will lead to a better environment for the use of HCCs. Figure 6 presents the relation between (γ) and the yearly tuition. Under current conditions, the average tuition in private universities exceeds the tuition that would make HCCs able to finance these programs without external aid.

Further easing of the parameters and variables may lead to the viability of HCCs. Unemployment was assumed to be constant and the same for the group of graduates from HEIs and for the total population. Accordingly, the default rate has been conservatively set higher than the proportion of graduates affiliated to the Social Security System. Following Palacios (op. cit), administration costs were set at 2 %, assuming a critical mass of students to spread the fixed costs among a greater number of students. Any change on these variables affects the value of HCC, as presented in Figure 7.

Figure 6. (γ) – Yearly Tuition (USD)

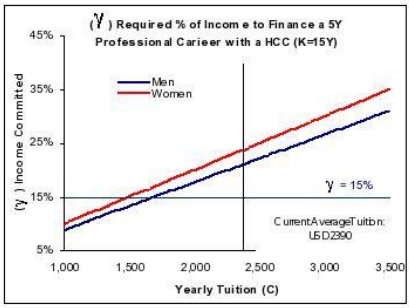
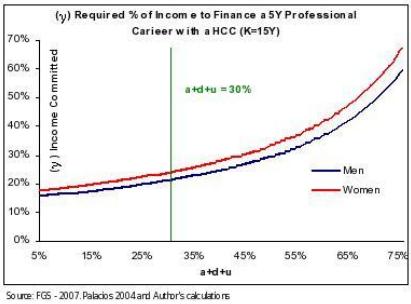


Figure 7. (γ) – (a, u y d)



The sensitivity analysis shows that HCC implementation needs further adjustments than go beyond changes in just one of the variables. But any improvement in measuring education returns, or any effort to improve macro economical stability affecting the above mentioned variables are steps towards HCCs implementation.

7. Conclusions

The present article follows Palacios' (op. cit.) model for Human Capital Contract Valuation in order to estimate the returns to education and to assess the viability of such contracts in Colombia. It is found that financing students at the professional level in private universities can only be partially done without any Government support. Other findings suggest that incentives exist for private investors to totally support students willing to pursue studies in public institutions. The latter might not be socially desirable as the current demand pressure on the public HEIs is high: the lack of capacity leaves out 80 % of the aspirants every year.

HCC still can be used to redirect resources and support students who wish to access private HEIs with governmental aid. The Government can be a guarantor and subsidize a part of the contract, at least at the beginning of their implementation. This form of subsidy would be efficient as it would increase competition among universities and also because it would avoid rigidities that are present in public institutions.

With regard to the estimation of graduates' income, the present article used data from the Following Graduates Survey 2007 (FGS 2007). Focusing on the group of Higher Education graduates, income forecasts were estimated by two model specifications (Modified Mincerian Model and Splines Model) and each one, through two econometric alternatives (OLS/RSE and IR), according to the previous literature (Mincer, 1974; Núñez and Sánchez, 2000; Low, et al. 2004; Daniels and Rospabé, 2005; Weldi, 2007; Forero and Ramírez 2008; and García et al., 2009). As expected, returns to education are greater in relation to higher levels of education. The income gap among genders seems smaller than

in the previous analysis for Colombia, at least at higher education levels. The geographic residence of the graduates affects the outcome of income: it favors the performance of income in the capital city or abroad. The field of study also determines the level of income after graduation in favor of students from Law and Engineering over Economics.

There are some issues that earning equations have not been able to solve and that require further study. Among those issues is the relationship between income growth and Age/Experience, and the link between current wages in different industries and the expected future income of a student. Furthermore, these issues should be considered dynamically.

It is important to consider the effect of the conservative measures that were taken on the econometric models and in HCCs valuation, and their downward influence on the final outcomes. Reconsidering some of the parameters brings the possibility of HCCs implementation closer.

Furthermore, the FGS 2007 is a new database. The fact that there are still only two waves of this survey does not allow the model to take into account dynamic effects on the returns to higher education. Future studies will be able to analyze such effects by using the same database. Further research is required for model specification and for better collection of information that would allow a better estimation of the students who outperform the average.

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Appendix

A.1 Robustness Exercises

A.1.1 Wage and Non Earnings Observations

Table A.1 includes outcome for Mincerian Model Estimations and Table A.2 presents the Splines Model estimations. For the wage earning observations the effects' direction and significance of the variables remain mostly unchanged. Regression Coefficient of Determination is higher than for any of the estimations made with the complete sub sample. Still, it can be seen the difficulty the models have to fit the data for the non wage earning observations. The direction and significance degree of several variables change when this model is considered.

A.2 Forecasting Income at Graduation

For both models, Mincerian and Splines, two different set of forecasts are presented: the first using OLS and RR outcomes, and the latter using IR estimates. The estimation for groups of students to form HCC securities is presented for the Economics and the fields of study that are increase income over the reference.The age at graduation is set at 23 years which is lower than the mean observed. The above is a conservative measure as the regression prizes older graduates. This might reward highly experienced students, but it is not necessarily true for students who had problems in their academic performance and delayed graduation. Furthermore, the parental education level used was the mean of the sub sample for each parent. Finally, taking into account the mode for working hours (49 % in the 31-48h/weekly interval), a level of 160 hours monthly is chosen. Above descriptions account for the hypothetical means of vectors, X_h, from a group of $m = 10$ students.

For the case of OLS/RSE, to calculate the confidence interval of the mean prediction, given the size of the sub sample, the Central Limit Theorem is summoned to assume that the income mean is normally distributed and the transformed variance covariance matrix, $\hat{\Omega}\,(b_{ij})^2_{rob}$, is used to calculate confidence intervals:

$$\log \hat{\bar{y}}_h \pm t\,(1-\alpha/2; N-k)\,\hat{\sigma}_{\bar{y}} \tag{10}$$

where

$$\hat{\sigma}^2_{\bar{y}} = MSE\left(\frac{1}{m} + X'_h \hat{\Omega}\,(b_{ij})\,X_h\right) \tag{11}$$

For the IR's, it is assumed that the mean of income is normally distributed as well. The estimation includes a new term σ_{IR} from Eq.(1), analogous to the MSE from OLS. Confidence intervals lose a degree of freedom accounting for the estimation of $\hat{\sigma}_{\text{IR}}$; additionally, the hypothetical vector of explanatory variables carries an extra zero with the expected value of the error, and the estimated covariance matrix comes from the inverse of the second derivatives of the log likelihood function, L, with respect to the parameters, V, with $(k+1)\times(k+1)$ dimensions (Maddala, 1983). Accordingly:

$$\log \hat{\bar{y}}_h \pm t\,(1-\alpha/2; N-k-1)\,\hat{\sigma}_{\bar{y}} \tag{12}$$

where

$$\hat{\sigma}^2_{\bar{y}} = \hat{\sigma}_{IR}\left(\frac{1}{m} + X'_h V X_h\right) \tag{13}$$

The point estimates, from Eq.(4) and Eq.(5), for $\log \hat{\bar{y}}_h$ are transformed in their respective value in Colombian Pesos (COP) and USD and presented in Table A.5, with the outcome for private universities students, and in Table A.6 with the result for their public counterparts.

Differences between the incomes of the graduates are important considering the effect of the field and the gender. While a male engineer is expected to have an annual income over USD10,000,[6] the expected income for a female economist would be below USD8,500 regardless of the method used in the prediction. For reference, the GDP per capita in Colombia was USD3,611.47 in 2007 according to the IMF estimations.[7] Thus, income for graduates at the professional level in the fields of Economics and Engineers represent, in average, between 2.5 to 3.5 times the income of the Colombian GDP per capita.

[6] Annual income calculated over 14 working months.

[7] From the IMF webpage: Query on the World Economic Outlook Database, April 2008. Information available at: http://www.imf.org/external/pubs/ft/weo/2008/

Table A.1 Mincerian Model. Wage vs. Non Wage Earning Observations

Wage Earning Observations						Wage Earning Observations					
Obs 17140		OLS	RSE		IR**	Obs 2574		OLS	RSE		IR**
Parameter	Estimate	Std Error	Rob Std Err	Estimate	Std Error	Parameter	Estimate	Std Error	Rob Std Err	Estimate	Std Error
Intercept	7,0719	0,1223	0,1279	6,8333	0,1277	Intercept	7,1050	0,4028	16,6370	6,8610	0,4087
yrs_edu	0,2788	0,0055	0,0056	0,2877	0,0057	yrs_edu	0,2710	0,0193	14,1556	0,2821	0,0196
age	0,0744	0,0055	0,0056	0,0791	0,0057	age	0,0632	0,0161	3,8603	0,0690	0,0162
age_Sq	-0,0008	0,0001	0,0001	-0,0008	0,0001	age_Sq	-0,0007	0,0002	-2,9688	-0,0007	0,0002
gender	0,1241	0,0084	0,0084	0,1413	0,0087	gender	0,1261	0,0271	4,6616	0,1439	0,0272
ln(h_ month)	0,2006	0,0086	0,0086	0,1980	0,0090	ln(h_ month)	0,2448	0,0216	9,6313	0,2350	0,0219
ed_madr	0,0135	0,0016	0,0016	0,0141	0,0017	ed_madr	0,1082	0,0429	2,5574	0,0972	0,0432
ed_padr	0,0223	0,0015	0,0015	0,0236	0,0016	ed_padr	0,0118	0,0049	2,3927	0,0127	0,0049
priv_HEI	0,0492	0,0124	0,0124	0,0523	0,0130	priv_HEI	0,0154	0,0052	2,9937	0,0165	0,0052
Agronom	-0,0365	0,0379	0,0379	-0,0513	0,0396	Agronom	-0,2485	0,1000	-2,9059	-0,2731	0,1023
Arts	-0,1995	0,0223	0,0223	-0,2381	0,0233	Arts	-0,2914	0,0524	-5,4876	-0,3135	0,0527
Educatn	-0,3759	0,0187	0,0187	-0,3820	0,0196	Educatn	-0,5185	0,0810	-5,6485	-0,4948	0,0833
Law	0,1087	0,0197	0,0197	0,0958	0,0203	Law	0,0266	0,0446	0,5978	0,0163	0,0445
Healt	0,0541	0,0178	0,0178	0,0494	0,0184	Healt	-0,0777	0,0596	-1,3064	-0,0767	0,0598
SociSt	-0,1455	0,0138	0,0138	-0,1690	0,0144	SociSt	-0,2391	0,0440	-5,4636	-0,2517	0,0442
Engineer	0,0658	0,0102	0,0102	0,0574	0,0106	Engineer	-0,0131	0,0359	-0,3693	-0,0175	0,0360
Math	-0,0826	0,0298	0,0298	-0,0964	0,0310	Math	-0,1755	0,1099	-1,8086	-0,2178	0,1100
R Sq		0.379411		0.3354(u)		R Sq		0.2702		0.2351(u)	
Adj R Sq		0.2613		0.2460(l)		Adj R Sq		0.3783		0.3629(l)	
F Value	337.40		$Pr > F$	$< ,0001$		F Value	30.35		$Pr > F$	$< ,0001$	

* Not significant at the 5 % confidence level

** Pseudo R Squared for the IR is the Squared Multiple Correlation

Source: FGS–2007 and author Calculations

Table A.2 Splines Model. Wage vs. Non Wage Earning Observations

Wage Earning Observations						Wage Earning Observations					
Obs 17140		OLS	RSE		IR**	Obs 2574		OLS	RSE		IR**
Parameter	Estimate	Std Error	Rob Std Err	Estimate	Std Error	Parameter	Estimate	Std Error	Rob Std Err	Estimate	Std Error
Intercept	11,4091	0,1075	0,1152	11,2635	0,1117	Intercept	11,4682	0,3119	0,3232	11,4184	0,3136
age	0,0853	0,0557	0,0059	0,0924	0,0059	age	0,0600	0,0164	0,0167	0,0640	0,0164
age_Sq	-0,0010	0,0001	0,0001	-0,0011	0,0001	age_Sq	-0,0006	0,0002	0,0002	-0,0006	0,0002
gender	0,1193	0,0097	0,0097	-0,1327	0,0100	gender	0,1204	0,0304	0,0300	0,1369	0,0304
ln(h_month)	0,1998	0,0086	0,0104	0,1968	0,0090	ln(h_month)	0,2442	0,0216	0,0253	0,2339	0,0219
ed_madr	0,0136	0,0016	0,0017	0,0143	0,0017	ed_madr	0,0156	0,0052	0,0051	0,0168	0,0052
ed_padr	0,0219	0,0015	0,0015	0,0235	0,0016	ed_padr	0,0121	0,0049	0,0049	0,0132	0,0049
priv_HEI	0,0501	0,0124	0,0123	0,0554	0,0130	priv_HEI	0,1066	0,0431	0,0427	0,0984	0,0434
dt_age2	0,0001	0,0000	0,0000	0,0001	0,0000	dt_age2	-0,0001	0,0001	0,0001	-0,0001	0,0001
dp_age2	0,0002	0,0000	0,0000	0,0002	0,0000	dp_age2	-0,0001	0,0001	0,0001	-0,0001	0,0001
dt_gen	-0,0249	0,0275	0,0290	-0,0484	0,0290	dt_gen	-0,0317	0,0957	0,0967	-0,0453	0,0980
dp_gen	0,0396	0,0218	0,0206	0,0680	0,0223	dp_gen	0,0478	0,0768	0,0745	0,0642	0,0763
Agronom	-0,0242	0,0379	0,0378	-0,0292	0,0396	Agronom	-0,2438	0,1006	0,0862	-0,2587	0,1026
Arts	-0,1975	0,0223	0,0225	-0,2311	0,0232	Arts	-0,2872	0,0528	0,0531	-0,3028	0,0530
Educatn	-0,3740	0,0188	0,0198	-0,3738	0,0197	Educatn	-0,5180	0,0812	0,0919	-0,4907	0,0834
Law	0,1204	0,0198	0,0186	0,1130	0,0203	Law	0,0249	0,0450	0,0440	0,0186	0,0448
Health	0,0567	0,0179	0,0180	0,0593	0,0185	Health	-0,0726	0,0600	0,0600	-0,0648	0,0601
SociSt	-0,1407	0,0141	0,0140	-0,1559	0,0146	SociSt	-0,2351	0,0446	0,0440	-0,2419	0,0447
Engineer	0,0743	0,0103	0,0103	0,0719	0,0107	Engineer	-0,0093	0,0362	0,0356	-0,0098	0,0362
Math	-0,0690	0,0299	0,0314	-0,0047	0,0310	Math	-0,1700	0,1106	0,0976	-0,2011	0,1105
tech	-0,0459	0,0025	0,0025	-0,0422	0,0026	tech	-0,0292	0,0083	0,0088	-0,0275	0,0084
posgrad	0,1387	0,0178	0,0154	0,1494	0,0184	posgrad	0,3234	0,0747	0,0706	0,3896	0,0748
R Sq		0.3828		0.3354(u)		R Sq		0.2709		0.2352(u)	
Adj R Sq		0.3815		0.3629(l)		Adj R Sq		0.2605		0.2399(l)	
F Value		294.64	$Pr > F$	$< ,0001$		F Value		26.18	$Pr > F$	$< ,0001$	

* Not significant at the 5 % confidence level

** Pseudo R Squared for the IR is the Squared Multiple Correlation

Source: FGS–2007 and author Calculations

Table A.3 Splines Model. Wage vs. Non Wage Earning Observations

	Gender	Method	Y(COP thds) 95 % Conf. Intervals			Y(COP thds) 95 % Conf. Intervals		
Economics, Finance, Accounting and Bussines Administration	M	Mincer/OLS	1,470,48	1,462,72	1,478,27	771,09	767,03	775,18
	M	Mincer/IR	1,497,91	1,487,27	1,508,63	785,48	779,9	791,1
	M	Splines/OLS	1,353,54	1,352,43	1,354,66	709,77	709,19	710,36
	M	Splines/IR	1,444,91	1,444,54	1,445,28	757,69	757,49	757,88
	F	Mincer/OLS	1,302,45	1,295,58	1,309,35	682,98	679,38	686,6
	F	Mincer/IR	1,300,53	1,291,29	1,309,84	681,98	667,13	686,86
	F	Splines/OLS	1,205,05	1,204,06	1,206,05	631,91	631,39	632,43
	F	Splines/IR	1,267,88	1,267,59	1,268,17	664,86	664,7	665,01
Health Sciences	M	Mincer/OLS	1,519,41	1,511,39	1,527,48	796,76	792,55	800,99
	M	Mincer/IR	1,573,77	1,562,58	1,585,04	825,26	819,39	831,17
	M	Splines/OLS	1,402,15	1,400,99	1,403,30	735,26	734,66	735,87
	M	Splines/IR	1,501,92	1,501,41	1,502,44	787,58	787,31	787,85
	F	Mincer/OLS	1,345,79	1,338,69	1,352,93	705,71	701,99	709,46
	F	Mincer/IR	1,366,39	1,356,68	1,376,18	716,51	711,42	721,65
	F	Splines/OLS	1,248,33	1,247,30	1,249,36	654,6	654,06	655,14
	F	Splines/IR	1,317,91	1,317,50	1,318,32	691,09	690,87	691,31
Engineering	M	Mincer/OLS	1,595,76	1,547,56	1,564,01	815,82	811,51	820,14
	M	Mincer/IR	1,586,41	1,575,14	1,597,76	831,89	825,98	837,84
	M	Splines/OLS	1,442,67	1,441,48	1,443,86	756,51	755,89	757,13
	M	Splines/IR	1,536,10	1,535,74	1,536,46	805,51	805,32	805,69
	F	Mincer/OLS	1,377,99	1,370,72	1,385,29	722,59	718,78	726,42
	F	Mincer/IR	1,377,37	1,367,58	1,387,22	722,27	717,14	727,44
	F	Splines/OLS	1,284,40	1,283,35	1,285,46	673,52	672,97	674,07
	F	Splines/IR	1,347,90	1,347,59	1,348,20	706,82	706,65	706,98
Law	M	Mincer/OLS	1,594,91	1,586,49	1,603,37	836,34	831,93	840,78
	M	Mincer/IR	1,648,51	1,636,79	1,660,32	864,45	858,3	870,65
	M	Splines/OLS	1,482,94	1,481,71	1,484,16	777,63	776,99	778,27
	M	Splines/IR	1,574,67	1,574,12	1,575,22	825,73	825,44	826,02
	F	Mincer/OLS	1,412,66	1,405,20	1,420,16	740,78	736,87	744,71
	F	Mincer/IR	1,431,29	1,421,11	1,441,54	750,54	745,21	755,92
	F	Splines/OLS	1,320,25	1,319,16	1,321,34	692,32	691,75	692,89
	F	Splines/IR	1,381,74	1,381,28	1,382,21	724,56	724,32	724,81

Source: FGS–2007 and author Calculations

Table A.4 Expected Monthly Income at Graduation. Public Universities

	Gender	Method	Y(COP thds) 95 % Conf. Intervals			Y(COP thds) 95 % Conf. Intervals		
Economics, Finance Accounting and Business Administration	M	Mincer/OLS	1,392,78	1,385,43	1,400,17	730,35	726,5	734,23
	M	Mincer/IR	1,421,59	1,400,49	1,431,76	745,46	740,16	750,79
	M	Splines/OLS	1,280,34	1,279,28	1,261,39	671,39	670,84	671,94
	M	Splines/IR	1,361,44	1,361,02	1,361,87	713,92	713,7	714,14
	F	Mincer/OLS	1,233,63	1,227,12	1,240,17	646,9	643,48	650,33
	F	Mincer/IR	1,234,26	1,225,50	1,243,09	647,23	642,63	651,86
	F	Splines/OLS	1,139,88	1,138,94	1,140,82	597,74	597,24	598,23
	F	Splines/IR	1,194,64	1,194,29	1,194,99	626,45	626,27	626,64
Health Sciences	M	Mincer/OLS	1,439,13	1,431,53	1,446,77	754,66	750,67	758,67
	M	Mincer/IR	1,493,58	1,482,95	1,504,28	783,21	777,64	788,82
	M	Splines/OLS	1,326,31	1,325,22	1,327,41	695,5	694,92	696,07
	M	Splines/IR	1,415,16	1,414,64	1,415,69	742,09	741,81	742,36
	F	Mincer/OLS	1,274,69	1,267,96	1,281,45	668,42	664,9	671,97
	F	Mincer/IR	1,296,77	1,287,55	1,306,05	680	675,17	684,87
	F	Splines/OLS	1,180,81	1,179,84	1,181,79	619,2	618,69	619,71
	F	Splines/IR	1,241,78	1,241,35	1,242,21	651,17	650,94	651,4
Engineering	M	Mincer/OLS	1,473,56	1,465,78	1,481,38	772,71	768,63	776,81
	M	Mincer/IR	1,505,57	1,494,88	1,516,35	789,5	783,89	795,15
	M	Splines/OLS	1,364,64	1,363,52	1,365,77	715,6	715,01	716,19
	M	Splines/IR	1,447,34	1,446,94	1,447,80	758,98	758,75	759,2
	F	Mincer/OLS	1,305,18	1,298,29	1,312,10	684,41	680,8	688,04
	F	Mincer/IR	1,307,18	1,297,90	1,316,53	685,47	680,6	690,37
	F	Splines/OLS	1,214,94	1,213,94	1,215,94	637,09	636,57	637,62
	F	Splines/IR	1,270,04	1,269,66	1,270,41	665,99	665,79	666,18
Law	M	Mincer/OLS	1,510,64	1,502,66	1,518,66	792,15	787,97	796,36
	M	Mincer/IR	1,564,51	1,553,38	1,575,72	820,4	814,57	826,28
	M	Splines/OLS	1,402,73	1,401,58	1,403,89	735,57	734,96	736,18
	M	Splines/IR	1,483,71	1,483,12	1,484,30	778,03	777,72	778,34
	F	Mincer/OLS	1,338,02	1,330,95	1,345,12	701,64	697,93	705,36
	F	Mincer/IR	1,358,35	1,348,69	1,368,08	712,3	707,23	717,4
	F	Splines/OLS	1,248,85	1,247,82	1,249,88	654,88	654,34	655,42
	F	Splines/IR	1,301,93	1,301,43	1,302,43	682,71	682,45	682,97

Source: FGS–2007 and author Calculations

A.3 Deriving a Mincerian Solution

Eq.(A.5) links the Mincerian Earnings function with the PIV of the graduates' lifetime earnings therefore graduates' PVI can be rewritten as:

$$PVI = e^{-is} \cdot Y_s \int_0^k e^{(h-i)t-gt^2} dt \tag{14}$$

Some algebraic manipulation will allow to reset the Eq.(A.5) into a known form of the Normal Cumulative Distribution Function (NCDF).[8]

$$PVI = Y_s \cdot e^{-is} \cdot e^{\frac{(h'-i)^2}{4g}} \int_0^k e^{(h'-i)t-gt^2-\frac{(h'-i)^2}{4g}} dt \tag{16}$$

where $h' = h - 2 \cdot A_{gr} \cdot g$ and A_{gr} is age at graduation.

Defining μ and σ for the NCDF

$$\mu = \frac{(h'-i)}{2g} \quad \text{and} \quad \sigma = \sqrt{\frac{1}{2g}} \tag{17}$$

Rearranging arguments and rewriting the NCDF after replacing μ and σ, Eq.(A.6) can be written as:

$$PVI = Y_s \cdot e^{-is} \cdot e^{\frac{(h'-i)^2}{4g}} \cdot \sqrt{\frac{\pi}{g}} \cdot (N(a) - N(b)) \ , \tag{18}$$

where

$$a = \sqrt{2g}\left(K - \frac{(h'-i)}{2g}\right) \quad \text{and} \quad b = \left(-\frac{(h'-i)}{2g}\right) \ . \tag{19}$$

Eq. (A.8) is the equation used in the valuation exercise of Palacios (2006, Chapter 6).

[8] Transformation comes from a change that makes models looking at the potential experience compatible with those models that use age as the proxy for determinants of income growth:

$$\begin{aligned} \ln Y &= \ln Y_0 + r \cdot s + h \cdot A - g \cdot A^2 \\ \ln Y &= \ln Y_0 + r \cdot s + h \cdot (A_{gr} + t) - g \cdot (A_{gr} + t)^2 \\ \ln Y &= \ln Y_0 + r \cdot s + h \cdot A_{gr} + h \cdot t - g \cdot A_{gr}^2 - 2 \cdot g \cdot A_{gr} \cdot t - g \cdot t^2 \\ \ln Y &= \ln Y_0 + r \cdot s + h \cdot A_{gr} - g \cdot A_{gr}^2 + (h - 2 \cdot g \cdot A_{gr}) \cdot t - g \cdot t^2 \end{aligned} \tag{15}$$