A NOTE ON THE THEORY OF THE FIRM UNDER UNCERTAINTY

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RESUMEN

En este artículo se muestra que el incremento en el nivel de riesgo de los precios reduce el nivel de producción óptimo en condiciones de aversión absoluta al riesgo creciente. Esto es, el impacto marginal del riesgo sobre el nivel de producción es independiente del tipo de aversión absoluta al riesgo (decreciente, constante o creciente).

Palabras clave: Incertidumbre de los precios, aversión al riesgo, teoría de la firma, incertidumbre.
Clasificación JEL: D8, D2.

ABSTRACT

We show that the increase in price riskiness reduces the optimal output under increasing absolute risk aversion. That is, the marginal impact of the risk on output is independent of the type of absolute risk aversion (decreasing, constant, or increasing).

Key Words: Price uncertainty, risk aversion, theory of the firm, uncertainty.
JEL Classification: D8, D2.

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A central result of the theory of the firm under uncertainty is that the increase in riskiness of the price reduces optimal output, given non-increasing absolute risk aversion (see Ishii, 1977). Thus, the marginal impact of risk on output is assumed ambiguous under increasing absolute risk aversion. This allows the possibility that the result is dependent on the type of risk aversion (increasing or non-increasing absolute risk aversion) or the functional form of the utility. This ambiguity became a stylised fact and none of the later studies including more sophisticated models such as Paroush and Wolf (1992), Dalal and Arshanapalli (1989, 1993), Lapan and Moschini (1994), Viaene and Zilcha 1998, Alghalith,M. and Dalal, A. (2002), and Alghalith (2003a, 2003b) attempted to remove this ambiguity even for special cases such as quadratic utility. This note shows this need not be the case. We show that the result is independent of the functional form and the type of risk aversion.

The standard model specifies the profit as $\pi = py - c(y)$ (see Sandmo, 1971), where p is the random output price, y is output, and c is the cost function. The random output price is given by $p = \bar{p} + \sigma\varepsilon$ where $\bar{p}$ is the expected price and σ is the standard deviation. The firm maximises the expected utility of the profit

$$\max_{y} EU(\pi),$$

where U is a Neumann-Morgensten utility function. To show the marginal impact of risk on the optimal output, standard comparative statics yield

$$\frac{\partial y^*}{\partial \sigma} = -\frac{y^* EU''(\pi^*)\left(p - c'(y^*)\right)\varepsilon + EU'(\pi^*)\varepsilon}{EU''(\pi^*)\left(p - c'(y^*)\right)^2 - c''(y^*)EU'(\pi^*)}, \qquad (1)$$

where the denominator is negative by the second-order condition of the maximisation problem. Ishii showed (1) to be negative under non-increasing absolute risk aversion. To show (1) is negative, we need to show $EU''(\pi^*)\left(p - c'(y^*)\right)\varepsilon \le 0.$

Proposition. $EU''(\pi^*)\left(p - c'(y^*)\right)\varepsilon \le 0$ if $U''' \le 0.$ [1]

Proof. $EU''(\pi^*)\left(p - c'(y^*)\right)\varepsilon = \left(\bar{p} - c'(y^*)\right)EU''(\pi^*)\varepsilon + EU''(\pi^*)\varepsilon^2.$ By the first-order condition $\left(\bar{p} - c'(y^*)\right) > 0$ and $EU''(\pi^*)\varepsilon = Cov\left(U''(\pi^*), \varepsilon\right) \le 0$ if $U'' \le 0.$

Clearly, the ambiguity remains for the case of $U'' > 0$ and increasing absolute risk aversion. However, our result serves the purpose of the paper; that is, it shows that Ishii's result does not change under certain forms of increasing absolute risk aversion.

[1] $U'' \le 0$ implies increasing absolute risk aversion.

Therefore, we can argue that the result is independent of the type of risk aversion, otherwise the sign of (1) will be reversed if $U^{"} \leq 0$. Furthermore, our assumption includes the quadratic (mean-variance) utility which is the most prominent and widely used function theoretically and empirically (see Paroush and Wolf, 1992, Rolfo, 1980, and Newbery and Stiglitz, 1981). It also includes other prominent functional forms such as Pope's (separable) utility, the cubic utility, higher order polynomial functions, and the expo- power utility.[2] This makes the result particularly useful for empirical research since the theory did not provide empirical researchers with theoretical criteria against which to test their empirical findings.

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[2] The cubic utility, higher order polynomial functions, and the expo- power utility fulfil the assumption $U^{"} < 0$ under certain parametric restrictions and an upper bound on the support induced on the profits.

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